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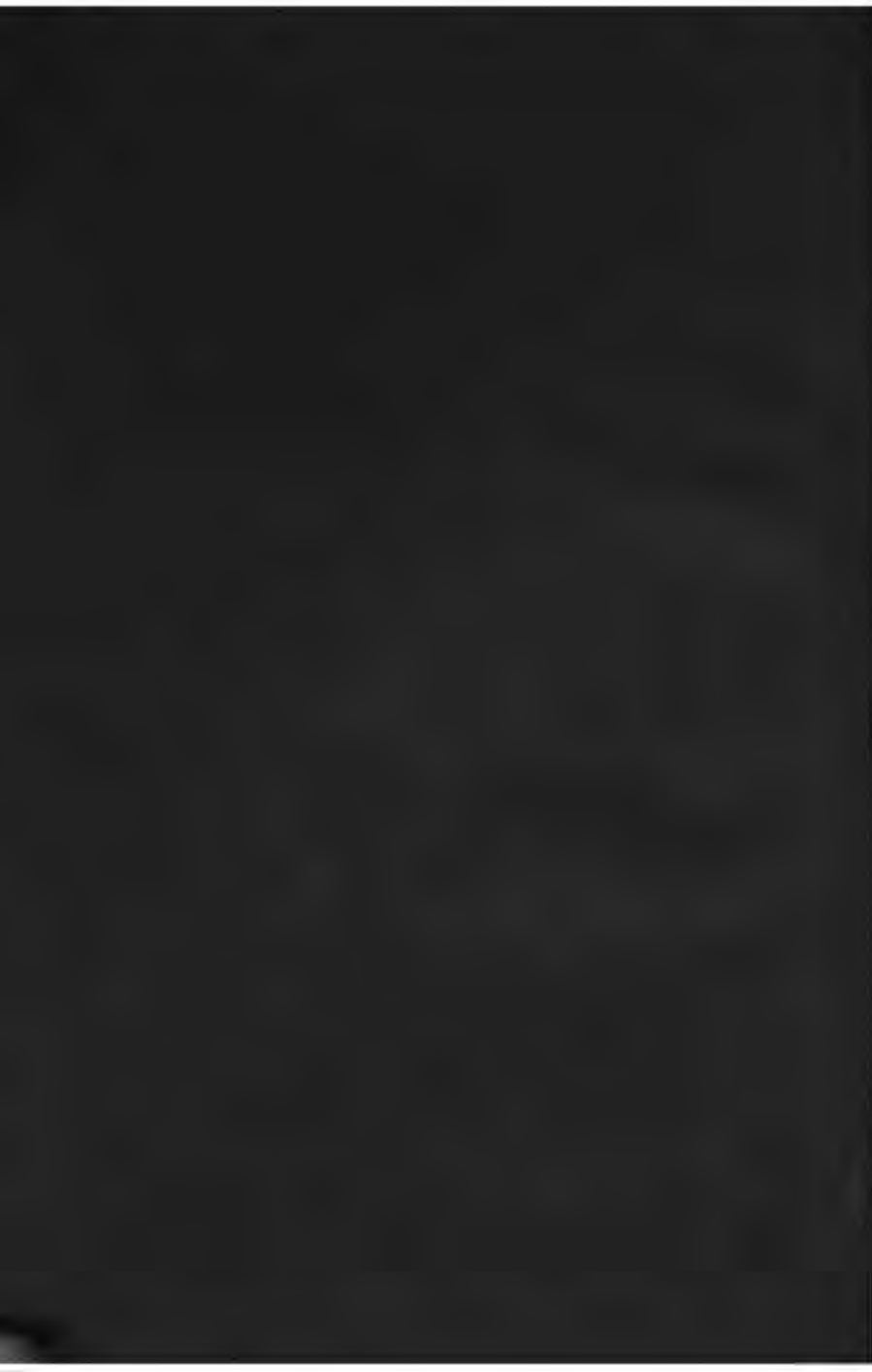
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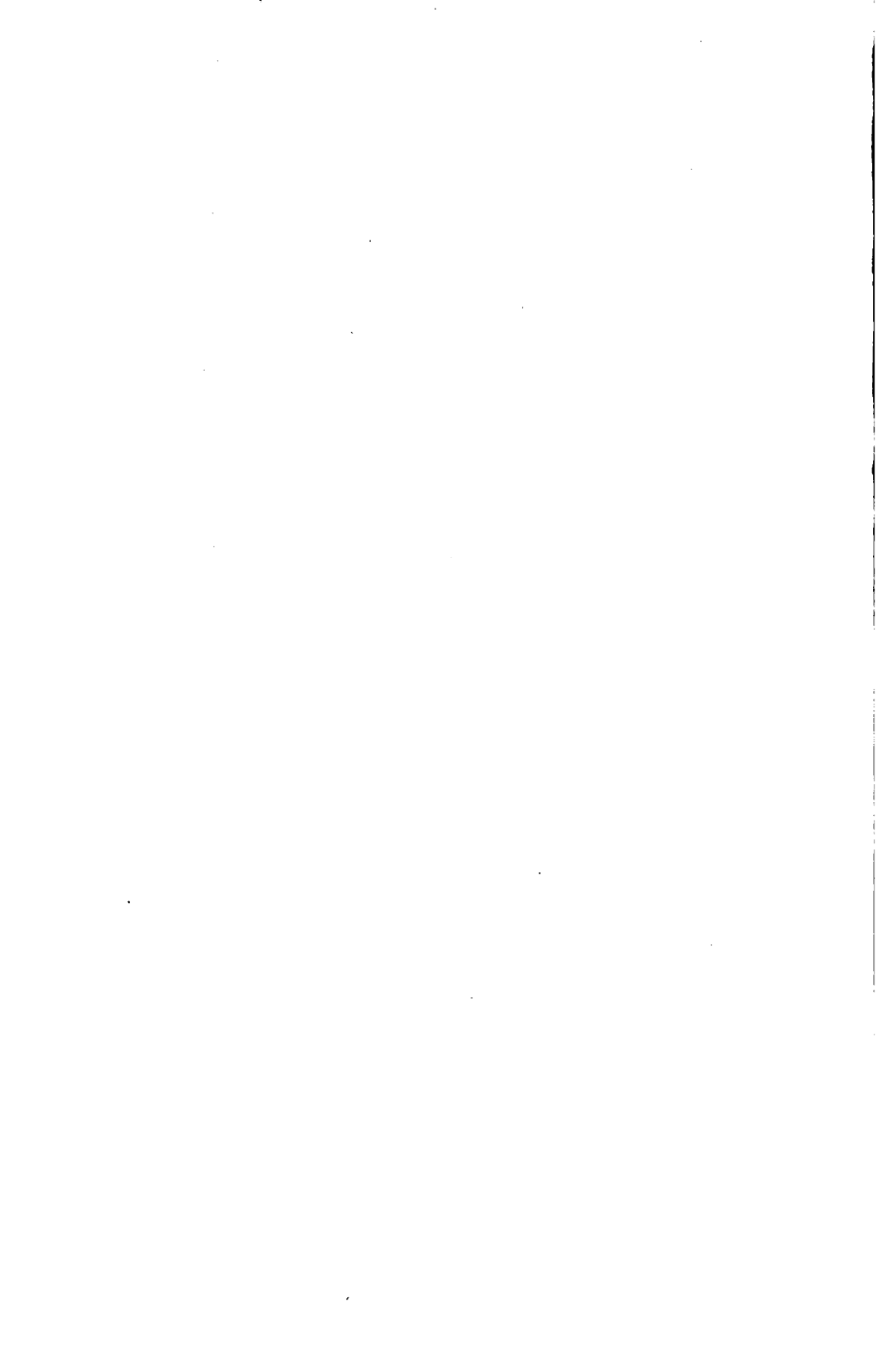
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# RETAINING-WALLS FOR EARTH.

INCLUDING

*THE THEORY OF EARTH-PRESSURE  
AS DEVELOPED FROM THE  
ELLIPSE OF STRESS.*

WITH

AN APPENDIX PRESENTING THE THEORY OF  
PROF. WEYRAUCH.

BY

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**Second Edition, Revised and Enlarged.**

NEW YORK:

JOHN WILEY & SONS,

53 EAST TENTH STREET.

1891.

Eng 618 .91

1892, Oct. 24.  
Scientific School.

58-19-  
N.Y.

JUN 20 1917  
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ROBERT DRUMMOND,  
Electrotyper,  
444 & 446 Pearl Street,  
New York.

FERRIS BROS.,  
Printers,  
326 Pearl Street,  
New York.

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## PREFACE.

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THE first edition of this work was based upon the theory advanced by Prof. Weyrauch in 1878, but owing to the length of the demonstrations used by him, it was thought advisable to present different and shorter demonstrations in this edition. To show that the new demonstrations give identical results with those obtained by Prof. Weyrauch, his demonstrations have been given in an appendix as they appeared in the first edition.

The new demonstrations are based upon the theory first advanced by Prof. Rankine in 1858. Those readers who are familiar with Rankine's *Ellipse of Stress* can omit pages 27 to 35, inclusive, in following the demonstrations.

An attempt has been made to present the theory in a shape easily followed by those who have only a knowledge of algebra, geometry, and trigonometry; whenever calculus has been resorted to, the work has been simplified as much as possible. For convenience in practice, the formulas have been arranged in a condensed shape in Part I, and are followed by numerous examples illustrating their application.

The values of various coefficients have been computed and tabulated and will be found to very materially decrease the labor of substitution in the formulas.

It is hoped that the introduction of a brief treatment of the supporting power of earth in the case of foundations, as well as the formula for determining the breadth of the base of a retaining-wall, will prove acceptable.

For valuable help in the verification of proofs of formulas, and the critical reading of the whole text, I acknowledge the kind assistance of Prof. Thos. Gray.

M. A. H.

TERRE HAUTE, IND., March, 1891.

## NOMENCLATURE.

---

- $\phi$  = the angle of repose, or the maximum angle which any force acting upon any plane within the mass of earth can make with the normal to the plane.
- $\epsilon$  = the angle made by the surface of the earth with the horizontal;  $\epsilon$  is *positive* when measured *above* and *negative* when measured *below* the horizontal.
- $\alpha$  = the angle which the back of the wall makes with the vertical passing through the heel of the wall;  $\alpha$  is *positive* when measured on the *left* and *negative* when measured on the *right* of the vertical.
- $\delta$  = the angle which the direction of the resultant earth-pressure makes with the horizontal.
- $\phi'$  = the angle of friction between the wall and its foundation.
- $\phi''$  = the angle of friction between the back of the wall and the earth.
- $H$  = the vertical height of the wall in feet.
- $h$  = the depth of earth in feet which is equivalent to a given load placed upon the surface of the earth.
- $B'$  = the width in feet of the top of the wall.
- $B$  = the width in feet of the base of the wall.
- $Q$  = the distance in feet from the toe of the wall to the point where  $R$  cuts the base.

$P$  = the resultant earth-pressure in pounds against a vertical wall.

$E$  = the resultant earth-pressure in pounds against any wall.

$R$  = the resultant pressure in pounds on the base of the wall.

$G$  = the total weight in pounds of material in the wall.

$\gamma$  = the weight in pounds of a cubic foot of earth.

$W$  = the weight in pounds of a cubic foot of wall.

$p$  = the intensity of the pressure in pounds on the base of the wall at the toe.

$p'$  = the intensity of the pressure in pounds on the base of the wall at the heel.

$p_0$  = the average intensity of the pressure in pounds on the base of the wall.

$x = H \tan \alpha$ .

# RETAINING-WALLS FOR EARTH.

## FORMULAS FOR EARTH-PRESSURE.

IN the following formulas  $\alpha$  and  $\epsilon$  are considered as positive, and the wall is assumed to be one foot long.

CASE I. *General case of inclined earth-surface and inclined back of wall.*

$$E = \frac{H^2 \gamma \cos(\epsilon - \alpha)}{2 \cos^2 \alpha \cos \epsilon} \times \sqrt{\sin^2 \alpha + \cos^2(\epsilon - \alpha) \left\{ \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \right\}^2 + 2 \sin \epsilon \sin \alpha \cos(\epsilon - \alpha) \left\{ \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \right\}}; \quad (1)$$

or

$$E = \frac{H^2 \gamma}{2} (B) \sqrt{(C) + (D)A^2 + (E)A}. \quad (1')$$

$$\tan \delta = \frac{\sin \alpha \cos \epsilon + \sin \epsilon \cos(\epsilon - \alpha)A}{\cos \epsilon \cos(\epsilon - \alpha)A}; \quad (1a)$$

or  $\tan \delta = \frac{\sin \alpha}{\cos(\epsilon - \alpha)A} + \tan \epsilon, \quad \dots \dots (1'a)$

where

$$A = \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \dots \quad (d)$$

CASE II. *Surface of earth inclined and  $\alpha = 0$ .*

$$E = P = \frac{H^2 \gamma}{2} \left\{ \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} = A \right\}. \quad (2)$$

From Diagram I the values of  $A$  can be found for all values of  $\phi$  from  $0^\circ$  to  $90^\circ$  and of  $\epsilon$  from  $0^\circ$  to  $90^\circ$ , varying by  $5^\circ$ .

$$\delta = \epsilon; \dots \dots \dots (2a)$$

*or for all vertical walls the direction of the earth-pressure is parallel to the surface of the earth.*

CASE III. *The surface of the earth parallel to the surface of repose.*

$$\epsilon = \phi.$$

$$E = \frac{H^2 \gamma}{2} \frac{\cos (\phi - \alpha)}{\cos^2 \alpha \cos \phi} \sqrt{\frac{\sin^2 \alpha + \cos^2 (\phi - \alpha)}{+ 2 \sin \alpha \sin \phi \cos (\phi - \alpha)}}. \quad (3)$$

$$\tan \delta = \frac{\sin \alpha + \sin \phi \cos (\phi - \alpha)}{\cos \phi \cos (\phi - \alpha)}. \dots (3a)$$

CASE IV. *The surface of the earth parallel to the surface of repose and the back of the wall vertical.*

$$\epsilon = \phi \quad \text{and} \quad \alpha = 0.$$

$$E = \frac{H^2 \gamma}{2} \cos \phi. \dots \dots \dots (4)$$

$$\delta = \phi. \dots \dots \dots (4a)$$

CASE V. *The surface of the earth horizontal.*

$$\epsilon = 0.$$

$$E = \frac{H^2 \gamma}{2} \sqrt{\tan^2 \alpha + \tan^2 \left( 45^\circ - \frac{\phi}{2} \right)}. \quad (5)$$

$$\tan \delta = \frac{\tan \alpha}{\tan^2 \left( 45^\circ - \frac{\phi}{2} \right)}. \quad (5a)$$

CASE VI. *The surface of the earth horizontal and the back of the wall vertical.*

$$\epsilon = 0 \quad \text{and} \quad \alpha = 0.$$

$$E = \frac{H^2 \gamma}{2} \tan^2 \left( 45^\circ - \frac{\phi}{2} \right). \quad (6)$$

$$\delta = 0. \quad (6a)$$

CASE VII. *Fluid pressure.*

$$\epsilon = \phi = 0.$$

$$E = \frac{H^2 \gamma}{2 \cos \alpha}. \quad (7)$$

$$\delta = \alpha. \quad (7a)$$

# GRAPHICAL CONSTRUCTIONS FOR DETERMINING THE THRUST OF EARTH.

The following constructions are perfectly general, and apply to *any plane* within a mass of earth. When applied

for determining the thrust of earth against a retaining-wall,  $\alpha$  and  $\epsilon$  are taken as *positive*.

\* *Construction (a).*

Let  $BE$  represent the surface of the earth and  $BA$  the back of the wall. Draw  $AF$  parallel to  $BE$ , and at any point  $D$  in  $AF$  lay off  $DF$  equal to the vertical  $DE$ . Draw

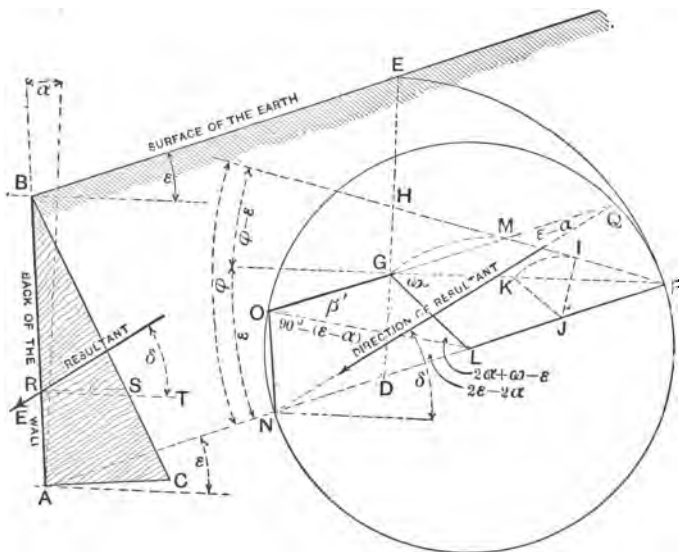


FIG. 1.

$FG$  horizontal, and  $FH$ , making the angle  $\phi$  with  $DF$ . With any point  $J$  in  $DF$  describe the arc  $KI$  tangent to  $HF$  at  $I$  cutting  $FG$  at  $K$ , and draw  $GL$  parallel to  $KJ$ ; with  $L$  as a centre and  $LF$  as radius, describe the circumference  $FQON$  cutting  $AD$  at  $N$ . Through  $N$  draw  $NO$

\* See "Theorie des Erddruckes auf Grund der neueren Anschauungen," by Prof. Weyrauch, 1881.

parallel to  $AB$  cutting the circumference  $FQON$  at  $O$ ; at  $A$  draw  $AC$  equal to  $OG$  and normal to  $AB$ ; the area of the triangle  $ABC$  multiplied by  $\gamma$  will be the thrust of the earth on the wall.

To determine the direction of the thrust  $E$ , prolong  $OG$  to  $Q$ ; then  $QN$  will be the direction of the thrust.

This thrust acts on the wall at  $\frac{2}{3}AB$  below  $B$ .

*\* Construction (b).*

Let  $BQ$  represent the surface of the earth, and  $BA$  the back of the wall. Draw  $AD$  parallel to  $BQ$ , and at any

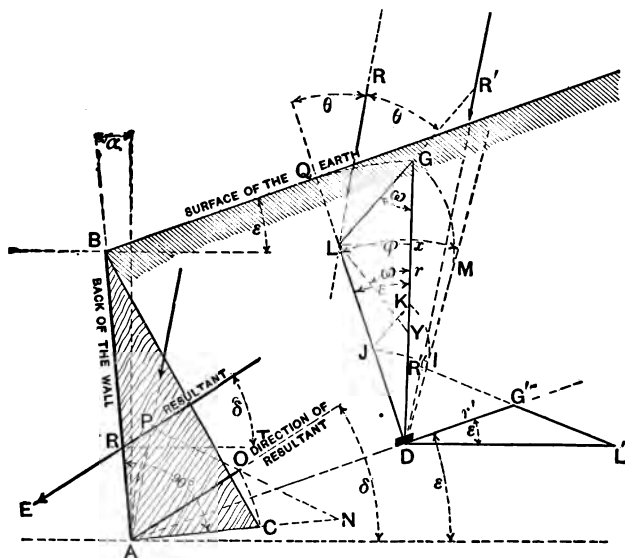


FIG. 2.

point  $D$  in  $AD$  draw the vertical  $DG$  equal to the normal  $DQ$ ; draw  $DM$  making the angle  $\phi$  with the normal  $DQ$ .

\* This construction follows directly from Rankine's Ellipse of Stress. See Rankine's Applied Mechanics.

At any point  $J$  in  $DQ$  as a centre, describe the arc  $IK$  tangent to  $DM$  cutting  $DG$  at  $K$ , and draw  $GL$  parallel to  $JK$ . Bisect the angle  $QLG$ , and at  $A$  draw  $AP$  parallel to  $LR$ . At  $A$  draw  $AN$  normal to  $AB$  and equal to  $DL$ ; with  $N$  as a centre and  $AN$  as radius, describe an arc  $AP$  cutting  $AP$  at  $P$ ; connect  $P$  and  $N$ , and make  $NO$  equal to  $LG$ ; with  $A$  as a centre and  $AO$  as a radius, describe the arc  $OC$  cutting  $AN$  at  $C$ ; then the area of the triangle  $ABC$  multiplied by  $\gamma$  will be the thrust against the wall. The direction of this thrust is parallel to  $AO$  and it is applied at  $\frac{2}{3}AB$  below  $B$ .

The constructions (a) and (b) give identical results in every case.

#### TRAPEZOIDAL AND TRIANGULAR WALLS.

Formulas for the width of the base of trapezoidal walls under the condition that the resultant  $R$  cuts the base at a point distant from the toe of the wall equal to one third the width of the base, or  $Q = \frac{1}{3}B$ .

CASE I. *The general case in which the back of the wall is inclined, and  $E$  makes an angle with the horizontal.*

$$B^2 + B \left( \frac{4E}{HW} \sin \delta + B' - x \right) = \frac{2E}{HW} (H \cos \delta + x \sin \delta) + 2B'x + B'' \quad (8)$$

CASE II. *The back of the wall vertical.*

$$x = 0.$$

$$B^2 + B \left( \frac{4E}{HW} \sin \delta + B' \right) = \frac{2E}{W} \cos \delta + B'' \quad (9)$$

CASE III. *The back of the wall vertical and the thrust normal to the wall.*

$$x = 0 \quad \text{and} \quad \delta = 0.$$

$$B^2 + BB' = \frac{2E}{W} + B'^2. \quad . \quad . \quad . \quad (10)$$

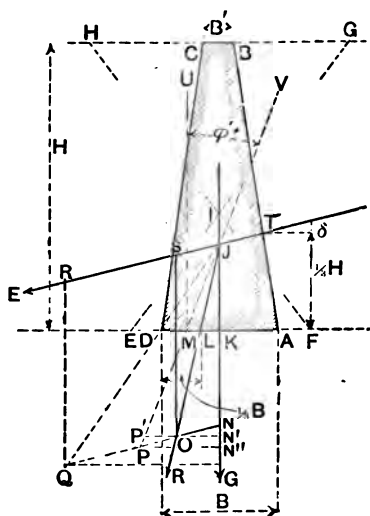


FIG. 3.

If  $B = B'$  and  $x = 0$ , the section of the wall is a rectangle, and (9) becomes

$$B^2 + B \frac{4E}{HW} \sin \delta = \frac{2E}{W} \cos \delta, \quad . \quad . \quad . \quad (9a)$$

and (10) becomes

$$B = \sqrt{\frac{2E}{W}}. \quad . \quad . \quad . \quad (10a)$$

Formulas for the width of the base of triangular walls under the condition that the resultant  $R$  cuts the base at a point distant from the toe of the wall equal to one third the width of the base, or  $Q = \frac{1}{3}B$ .

CASE I. *The general case in which the back of the wall is inclined, and  $E$  makes an angle with the horizontal.*

$$B^3 + B \left( \frac{4E}{HW} \sin \delta - x \right) = \frac{2E}{HW} (H \cos \delta + x \sin \delta). \quad (11)$$

CASE II. *The back of the wall vertical.*

$$\alpha = 0.$$

$$B^3 + B \left( \frac{4E}{HW} \sin \delta \right) = \frac{2E}{W} \cos \delta. \quad . \quad . \quad (12)$$

CASE III. *The back of the wall vertical, and the thrust normal to the wall.*

$$x = 0 \quad \text{and} \quad \delta = 0.$$

$$B = \sqrt{\frac{2E}{W}}. \quad . \quad . \quad . \quad . \quad (13)$$

*The above formulas do not contain the condition that  $R$  shall not make an angle greater than  $\phi'$  with the normal to the base of the wall.*

From Fig. 3,

$$\tan \phi' \geq \frac{E \cos \delta}{G + E \sin \delta} = \tan LJK, \quad . \quad . \quad (14)$$

*which expresses the condition under which the wall will not slide,*

DEPTH OF FOUNDATIONS.

CASE I. *When the intensity of the pressure on the earth is uniform.*

Letting  $x'$  equal the depth of the foundation below the surface,

$$x' = \frac{p_0(1 - \sin \phi)^2}{(1 + \sin \phi)^2 \gamma - W(1 - \sin \phi)^2}, \quad \dots \quad (15)$$

when the weight of the foundation is included; and

$$x' = \left\{ \frac{1 - \sin \phi}{1 + \sin \phi} \right\}^2 \frac{p_0}{\gamma}, \quad \dots \quad (16)$$

when the weight of the foundation is not included.

$x'$  is the minimum depth to which the foundation must be extended for equilibrium. The actual depth should be based upon the minimum value which  $\phi$  is likely to have under any condition of the earth.

CASE II. *When the intensity of the pressure on the earth is uniformly varying.*

$$x' = \frac{p_0}{\gamma} \frac{(1 - \sin \phi)^2}{1 + \sin^2 \phi}, \quad \dots \quad (19)$$

where  $x'$  is the minimum depth to which the foundation must be extended for equilibrium;

$$x_0 = \frac{1}{3} \frac{\sin \phi}{1 + \sin^2 \phi}, \quad \dots \quad (20)$$

where  $x_0$  is the maximum distance from the centre of the base of the foundation to the point where the resultant pressure cuts the base of the foundation.

## ABUTTING POWER OF EARTH.

$$P = \frac{(x')^2 \gamma}{2} \frac{1 + \sin \phi}{1 - \sin \phi}, \quad . . . . (21)$$

where  $P$  represents the maximum resultant pressure which horizontal earth can resist, when  $P$  is applied against a vertical plane of the depth  $x'$ .

## APPLICATIONS.

The determination of the earth-pressure by the preceding formulas and graphical constructions is a very simple operation when the angle  $\phi$  has been determined or assumed. That care and judgment be used in assuming the value of  $\phi$  is very important, since a change of a few degrees in the value of  $\phi$  sometimes causes a large change in the value of  $E$ . An inspection of Diagram I shows that the value of the coefficient  $A$  increases very rapidly as  $\phi$  decreases.

When the earth to be retained contains springs, the bank must be thoroughly drained if it is to be retained by an economical tight wall; if it is not drained, the angle  $\phi$  will be likely to become very small as the earth becomes wet.

When the location of the earth to be retained is subjected to jars, the value of  $\phi$  will be decreased.

Hence, in assuming the value of  $\phi$ , the engineer must be sure that the value assumed will be the least value which, in his judgment, it is likely to have.

In constructing the wall the judgment and authority of the engineer must again be exercised in order that the wall be constructed as designed.

In all cases, to insure perfect drainage between the back

of the wall and the earth, numerous "weep-holes" should be provided in the body of the wall, or proper arrangements made to carry away the water at the base of the wall. To facilitate drainage, the backing resting against the wall should be sand or gravel.

In no case should water be permitted to get under the foundation of the wall, neither should the earth in front of the wall be allowed to become wet.

In cold localities the back of the wall near the top should have a large batter to prevent the frost from moving the top courses of stone. As a guard against sliding, the courses of the wall should have very rough beds. The strength of a wall is increased the nearer it approaches a monolith.

Care should be taken to have the foundation broad and deep enough to prevent sliding and upheaving of the earth in front. In clay the foundation should be deep, while in sand or gravel it may be broad and shallow.

The following examples illustrate the application of the formulas:

Ex. 1. Design a trapezoidal wall of sandstone, weighing 150 lbs. per cubic foot, having a width of 3 ft. on top, a height of 30 ft., and the back inclining forward  $5^\circ$ , to retain a bank of sand sloping upward at an angle of  $20^\circ$ .

*Data.*

$\gamma = 100$  lbs.,  $W = 150$  lbs.;  $\epsilon = 20^\circ$ ,  $\phi = 39^\circ$ ,  $\alpha = 5^\circ$ ;  
 $H = 30$  ft.,  $B' = 3$  ft.,  $x = 2.63$  ft.

1°. *Graphical determination of the values of  $E$  and  $\delta$ .*

The graphical solution of the problem is shown in Fig. 4, where  $E$  is found to equal 15,000 pounds.  $\delta$  lies between  $35^\circ$  and  $36^\circ$ .

2°. Algebraic determination of  $E$  and  $\delta$ .

$$E = \frac{H^2 \gamma}{2} (B) \sqrt{(C) + (D)A^2 + (E)A} \dots (1')$$

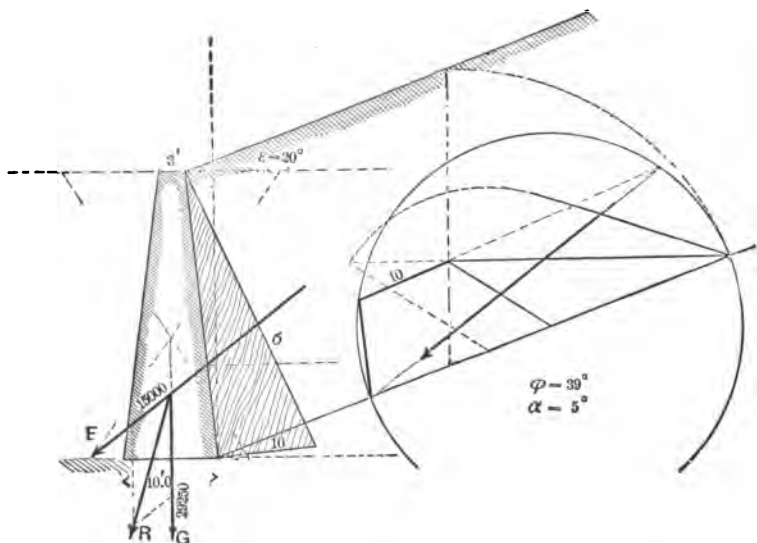


FIG. 4.

Substituting the values of  $B$ ,  $C$ ,  $D$ , and  $E$  as given in the tables, and that of  $A$  as given by Diagram I, this becomes

$$E = \frac{900 \times 100}{2} (1.036) \times \sqrt{(0.008) + (1.057)(0.264)^2 + (0.061)0.264},$$

$$E = 45,000 (1.036) \sqrt{0.098} = 14,500 \text{ lbs.}$$

$$\tan \delta = \frac{\sin \alpha}{\cos (\epsilon - \alpha) A} + \tan \epsilon, \dots (1' a)$$

$$\tan \delta = \frac{0.087}{0.966(0.264)} + 0.364,$$

$$\tan \delta = 0.705 = \tan 35^\circ 11', \text{ about.}$$

3°. *Algebraic determination of the value of B under the assumption that  $Q = \frac{1}{3}B$ .*

$$\begin{aligned} B^2 + B \left\{ \frac{4E}{HW} \sin \delta + B' - x \right\} \\ = \frac{2E}{HW} \left\{ H \cos \delta + x \sin \delta \right\} + 2B'x + B'^2. \quad (8) \end{aligned}$$

$$\begin{aligned} E^2 + B \left\{ \frac{4 \times 14500}{30 \times 150} 0.576 + 3 - 2.63 \right\} \\ = \frac{2 \times 14500}{30 \times 150} \{ 30 \times 0.817 + 2.63 \times 0.576 \} + 6 \times 2.63 + 9, \end{aligned}$$

$$B^2 + 7.79B = 172.53,$$

$$B = -3.89 \pm \sqrt{172.53 + 3.9^2};$$

$$\therefore B = 13.69 - 3.89 = 9.80 \text{ ft.};$$

or, practically, 10 feet is the required width of the base.

4°. *To determine if the wall will slide on a foundation of sandstone.*

From (14),

$$\tan \phi' > \frac{E \cos \delta}{G + E \sin \delta}.$$

$$\text{Taking } B = 10 \text{ ft., } G = \frac{10 + 3}{2} 30 \times 150 = 29250 \text{ lbs.}$$

$\delta = 35^\circ 11'$ ,  $\cos \delta = 0.817$ , and  $\sin \delta = 0.576$ , then

$$\frac{E \cos \delta}{G + E \sin \delta} = \frac{14500 \times 0.817}{29250 + 14500 \times 0.576} = 0.315.$$

From Table II, the value of  $\tan \phi'$  for masonry is 0.6 to 0.7; hence there is no danger of the wall sliding on the foundation.

5°. To determine the minimum depth to which the foundation must extend consistent with the stability of the earth.

First determine the maximum value of  $x_0$ . From (20),

$$x_0 = \frac{1}{3} \frac{\sin \phi}{1 + \sin^2 \phi},$$

where  $\phi$  must be assumed at its minimum value. Assume that the minimum value of  $\phi$  in this case is  $30^\circ$ ; then

$$x_0 = \frac{1}{3} \frac{0.577}{1.333} = 0.133,$$

showing that the resultant must cut the base of the foundation within 0.133 feet of its centre. The resultant cuts the base of the wall 1.67 feet from the centre of its base; hence the width of the foundation must be increased.

Assuming that the depth to which the foundation extends is 4 feet, and that it is vertical in the rear; then the direction of the resultant pressure (not including the additional weight of the foundation) will cut the base of the foundation 7.93 feet from the rear or heel. The required width of the base of the foundation is  $(7.93 - 0.13)2 = 15.6$ ; say, 16 feet.

The value of  $p_0$  can now be found, which corresponds to the assumed value of  $x' = 4$  feet.

From (19),

$$p_0 = x' \gamma \frac{1 + \sin^2 \phi}{(1 - \sin \phi)^2};$$

$$p_0 = 400 \frac{1.333}{0.179} = 2960 \text{ lbs.}$$

The average intensity of the pressure on the base of the foundation due to the resultant  $R$  is

$$\frac{29250 + 14500 \sin \delta}{16} = 2350 \text{ lbs.}$$

The foundation adds an intensity equal to  $4 \times 150 = 600$  pounds approximately; hence the actual value of  $p_0 = 2350 + 600 = 2950$  pounds; therefore, if the foundation has a depth of 4 feet and a base of 16 feet, the wall will not sink nor the earth in front of the wall heave, until  $\phi$  becomes less than  $30^\circ$ .

6°. *To determine if the wall and foundation will slide on the earth.*

This is resisted in two ways—by the friction between the masonry and the earth, and by a prism of earth in front of the wall.

The horizontal force tending to make the wall slide equals  $E \sin \delta$ , or  $14500.0.576 = 8352$  pounds. The horizontal force tending to make the foundation slide equals the resultant earth-pressure on the rear face of the foundation, which is vertical and 4 feet in height. From (6),

$$E = \left\{ \frac{(30 + 4)^2}{2} - \frac{30^2}{2} \right\} \gamma \tan^2 \left( 45^\circ - \frac{\phi}{2} \right),$$

or  $E = 12800 \times 0.226 = 2893.$

Then the total horizontal force tending to make the wall slide is

$$8352 + 2893 = 11245 \text{ lbs.}$$

From Table II the tangent of the angle of friction between masonry and moist clay is 0.33, which evidently is much smaller than the tangent of the actual angle of friction between masonry and dry earth. Assume this tangent to be 0.500.

The total vertical pressure upon the base of the foundation is 37600 pounds, hence the ability to resist sliding is  $37600 (0.5) = 18800$  pounds, which is much larger than 11245; hence there is no danger of the wall slipping, even if the earth in front of the wall does not act.

Ex. 2. Design a trapezoidal wall of sandstone weighing 150 lbs. per cubic foot, having a width of 3 ft. on top, a height of 30 ft., and the back inclining backward  $15^\circ$ , to retain a bank of sand sloping upward at an angle of  $30^\circ$ .

*Data.*

$\gamma = 100 \text{ lbs.}, W = 150 \text{ lbs.}; \epsilon = 30^\circ, \phi = 33^\circ, \alpha = -15^\circ;$   
 $H = 30 \text{ ft.}, B' = 3 \text{ ft.}, x = 8 \text{ ft.}$

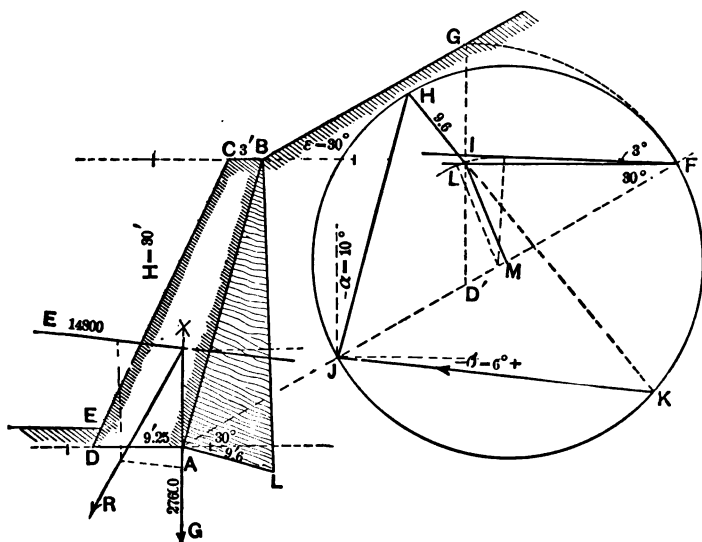
1°. *Graphical determination of the values of  $E$  and  $\delta$ .*

In Fig. 5, let  $EG$  represent the surface of the earth, and  $AB$  the back of the wall. Draw  $AF$  parallel to  $BG$ , and from any point  $D'$  in  $AF$  lay off  $D'F$  equal to the vertical  $D'G$ , and draw  $FL$  horizontal; lay off the angle  $IFD' = \phi = 33^\circ$ , and locate the point  $M$  in  $D'F$  so that if an arc be described with  $M$  as a centre and  $LM$  as a radius the arc will be tangent to  $IF$ ; then with  $M$  as a centre and  $MF$  as a radius, describe the circumference  $FHJ$  and draw  $JH$

parallel to  $AB$ ; at  $A$  draw  $AL$  perpendicular to  $AB$  and equal to  $HI$ . Then

$$\frac{(AB)(AL)}{2} \gamma = \frac{(30.9)(9.6)}{2} 100 = 14800 = E.$$

To determine  $\delta$ , prolong  $HI$  to  $K$  and draw  $KJ$ . Then the angle which this line makes with the horizontal is equal to  $\delta$ , which is  $6^\circ$  to  $7^\circ$  in this case.



**Fig. 5.**

### 2°. Algebraic determination of $E$ and $\delta$ .

Substituting in (1) and remembering that  $\alpha$  is negative,

$$E = 45000 (0.875) \sqrt{0.067 + 0.183 - 0.111} = 14600 \text{ lbs.}$$

From (1'a),

$$\tan \delta = \frac{-0.259}{0.707(0.524)} + .577 = -0.123 = \tan (-7^\circ).$$

3°. *Algebraic determination of the value of B under the assumption that  $Q = \frac{1}{3}B$ .*

Substituting the proper values in (11) and remembering that  $\alpha$  is negative,

$$B = -4.7 \pm \sqrt{163.44 + (4.7)^2} = 9.0 \text{ ft.}$$

The foundation can be designed in the manner outlined in Ex. 1.

Ex. 3. Determine the dimensions of a brick wall having a vertical back to retain a bank of sand sloping upward at an angle of  $20^\circ$ .  $\phi = 30^\circ$ ,  $H = 20'$ ,  $B' = 2'$ ,  $\gamma = 100$ .

1°. *Algebraic determination of E and  $\delta$ .*

Since  $\alpha = 0$ ,

$$E = \frac{H^2 \gamma}{2} A \dots \dots \dots (2)$$

$$E = \frac{400 \times 100}{2} 0.424 = 8480; \text{ say, } 8500 \text{ lbs.}$$

The value of  $A$  is readily found from Diagram I.

$$\delta = \epsilon = 20^\circ, \quad \text{since } \alpha = 0.$$

2. *Algebraic determination of the value of B under the condition that  $Q = \frac{1}{3}B$ .*

$$B^2 + B \left\{ \frac{4E}{HW} \sin \delta + B' \right\} = \frac{2E}{W} \cos \delta + B'^2. \quad (9)$$

From Table I,  $W = 125$  lbs. Then

$$B^2 + B \left\{ \frac{4 \times 8500}{20 \times 125} 0.342 + 2 \right\} = \frac{2 \times 8500}{125} 0.940 + 4,$$

or  $B^2 + 6.65B = 131.84.$

$$B = -3.36 \pm \sqrt{131.84 + 3.36^2},$$

and

$$B = -3.36 + 11.96 = 7.60 \text{ ft.}$$

Ex. 4. Determine the value of  $B$  in Ex. 3 under the assumption that  $\epsilon = 0$  (horizontal earth-surface).

$$E = \frac{H^2 \gamma}{2} \left\{ \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) = \frac{1 - \sin \phi}{1 + \sin \phi} \right\}, \quad (6)$$

or  $E = 20000 (0.333) = 6666$ , say 6700 lbs.

Since  $\alpha = 0$ , and  $\epsilon = 0$ ,  $\delta = 0$ ,

$$B^2 + BB' = \frac{2E}{W} + B'^2; \quad . \quad . \quad . \quad (10)$$

$$B^2 + 2B = 111.2;$$

$$B = -1 \pm \sqrt{111.2 + 1},$$

and

$$B = -1 + 10.59 = 9.6 \text{ ft.}$$

Ex. 5. Determine the value of  $B$  in Ex. 3, under the assumption that  $\epsilon = \phi = 30^\circ$ .

$$E = \frac{H^2 \gamma}{2} \cos \phi = 20000 (0.866) = 17320 \text{ lbs.}$$

From (9),

$$B^2 + B \left\{ \frac{4 \times 17320}{20 \times 125} 0.5 + 2 \right\} = \frac{2 \times 17320}{125} 0.866 + 4;$$

$$B^2 + 15.86B = 244.05;$$

$$B = -7.93 + \sqrt{244.05 + 7.93^2}.$$

and  $B = -7.93 + 17.52 = 9.6$  ft.

Ex. 6. Determine the resultant pressure against the back of a wall when the surface of the earth carries a load equivalent to 5 feet in depth of sand.

$H = 30$  ft.,  $\alpha = 10^\circ$ ,  $\phi = 30^\circ$ ,  $\epsilon = 0$ , and  $\gamma = 100$  lbs.

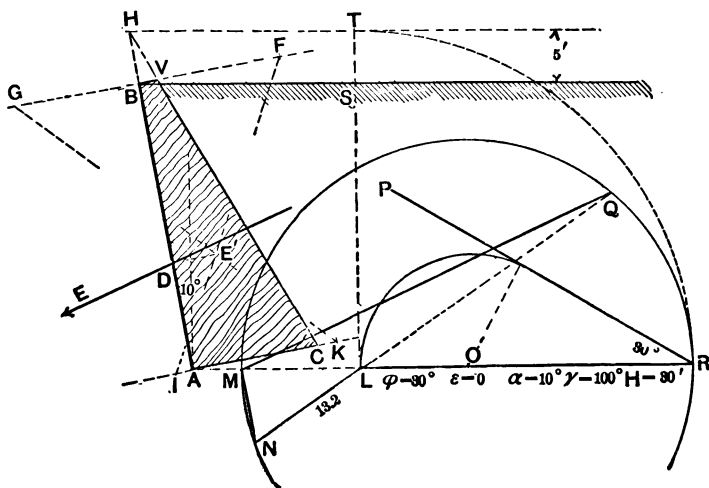


FIG. 6.

*Graphical solution of the problem.*—In Fig. 6, let  $BS$  represent the surface of the earth, and  $BA$  the back of the wall.

Make  $ST = 5$ , and draw  $HT$  and  $BH$ . Draw  $AR$  parallel to  $BS$ , parallel to  $HT$ , and make  $LR$  equal to  $LT$ ; lay off the angle  $LRP$  equal to  $30^\circ$ ; with  $O$  as a centre

draw an arc passing through  $L$  tangent to  $PR$ , and then with  $OR$  as a radius describe the circumference of the circle  $RQM$ , and at  $M$  draw  $MN$  parallel to  $AH$ ; at  $A$  and normal to  $AH$  draw  $AC$  equal to  $NL$ . Then

$$\frac{AC + BV}{2} BA \cdot \gamma = E.$$

The direction of  $E$  will be parallel to  $QM$ .

To determine the point of application of  $E$ , find the centre of gravity  $E'$  of  $ABVC$ , and draw  $E'D$  parallel to  $AC$ , then  $D$  will be the point of application of  $E$ .

$E'$  can be found as follows: Produce  $AC$  and  $BV$ , make  $AI = CK = BV$ ,  $BG = VF = AC$ , and join  $F$  and  $I$  and  $G$  and  $K$ . Then  $E'$ , the intersection of  $FI$  and  $GK$ , will be the centre of gravity of  $ABVC$ .  $BD$  can be found from the formula

$$BD \cos 10^\circ = \frac{1}{3} \frac{(TL)^3 - 3(TL)(TS)^2 + 2(TS)^3}{(TL)^3 - (TS)^3}.$$

See (30) of Appendix.

Ex. 7. Determine graphically the value of  $E$  when  $\epsilon = 0$  and  $\alpha = 0$ ,  $\phi$ ,  $\gamma$ , and  $H$  being given.

In Fig. 7 let  $BF$  represent the surface of the earth, and  $AB$  the back of the wall. Draw  $AL$  parallel to  $BF$  and make  $IL = IF$ ; lay off the angle  $GLH = \phi$ , and at any point  $K$  in  $LH$  draw  $MK$  perpendicular to  $HL$ , and lay off  $MO = MK$ ; draw  $MJ$  parallel to  $OI$ . Then will the arc  $IN$ , described with  $J$  as a centre and  $IJ$  as a radius, pass through  $I$  and be tangent to  $GL$ ; with  $J$  as a centre and  $JL$  as radius describe the circumference  $LH$ ; at  $A$  lay off  $AC = HI$  and normal to  $AB$ . Then

$$\frac{AC \times AB}{2} \gamma = E.$$

$E$  is parallel to  $BF$  and applied at  $D$ ,  $AD$  being equal to  $\frac{1}{3}AB$ .

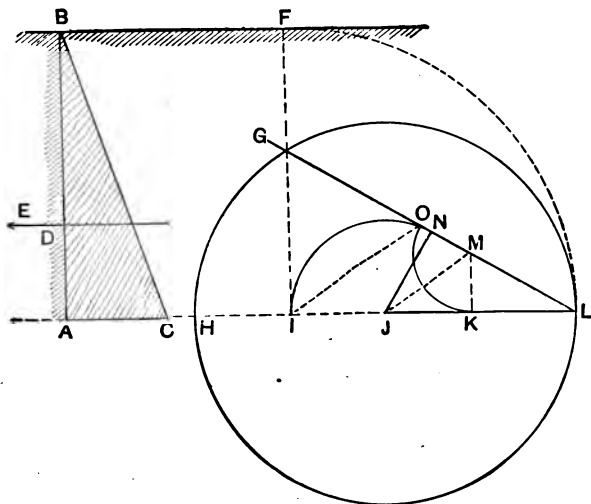


FIG. 7.

Ex. 8. Determine the earth-thrust on the profile shown in Fig. 8,  $H$ ,  $\gamma$ ,  $\phi$ , and  $\epsilon$  being given.

*Graphical solution of the problem.*—Let  $BCDEA$  represent the given profile, and let the surface of the earth be horizontal. Prolong  $BC$  until it intersects  $SA$  in  $S$ ; draw  $SR$  normal to  $BCS$  and equal to the intensity of the earth-pressure at  $S$ ; connect  $B$  and  $R$ . Then from the middle point of  $BC$  draw  $GF$  parallel to  $SR$ ; the distance  $GF$  multiplied by  $\gamma$  will be the average intensity of the earth-pressure on  $BC$ . In a similar manner the average intensities on  $CD$ ,  $DE$ , and  $EA$  can be found, and hence the total pressures on each determined. The points of application of these resultant pressures,  $E_1$ ,  $E_2$ ,  $E_3$ , and  $E_4$ ,

can be found by the method used in Ex. 6 for finding the centre of gravity of a trapezoid. The directions of

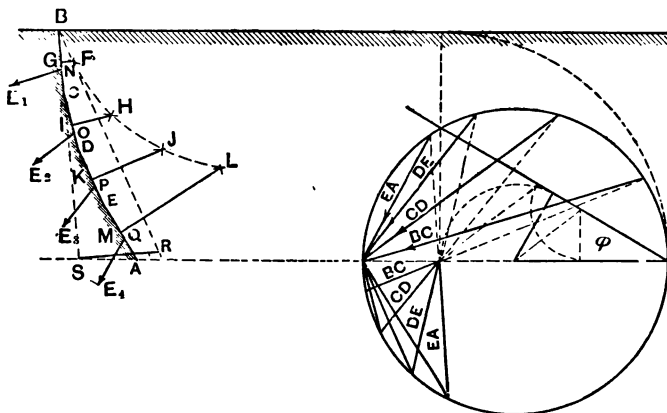


FIG. 8.

$E_1$ ,  $E_2$ ,  $E_3$ , and  $E_4$  are found from the construction on the right.

Ex. 9. Determine the thrust of the earth against a vertical wall when  $\epsilon$  is negative.

For the explanation of this construction, see Part II, page 47, Fig. 8a.

Ex. 10. From the following data determine  $E$ ,  $\delta$ , and  $Q$ :

$\epsilon = 0$ ,  $\phi = 38^\circ$ ,  $\alpha = 10^\circ 23'$ ;  $\gamma = 90$  lbs.,  $W = 170$  lbs.,

$H = 15$  ft.,  $B = 6$  ft.,  $B' = 2$  ft.

Ans.  $E = 3037$  lbs.,  $\delta = 27^\circ 13'$ ,  $Q = 2.2$  ft.

Ex. 11. Determine the dimensions of a trapezoidal wall built of dry, rough granite, having a vertical back and being 20 feet high, to safely retain the side of a sand cut,



$H = 15$  ft.,  $\alpha = 8^\circ$ ,  $\phi = 33^\circ 40'$ ,  $\gamma = 100$  lbs.,  $W = 170$  lbs.,  $B' = 3.5$  ft.

*Ans.*  $E = 5760$  lbs.,  $\delta = 18^\circ 7'$ ,  $B = 8$  ft.,  $Q = 2.7$  ft.

EX. 14. Determine  $E$ ,  $\delta$ ,  $B$ , and  $Q$ , when  $W = 170$  lbs.,  $\gamma = 100$  lbs.,  $\alpha = 8^\circ$ ,  $\epsilon = \phi = 33^\circ 40'$ ,  $H = 20$  ft.,  $B' = 2$  ft.

*Ans.*  $E = 21760$  lbs.,  $\delta = 32^\circ 25'$ ,  $B = 9$  ft.,  $Q = 3$  ft.

\* EX. 15. A wall 9 ft. high faces the steepest declivity of earth at a slope of  $20^\circ$  to the horizon; weight of earth 130 lbs. per cubic foot, angle of repose  $30^\circ$ . Determine  $E$  when  $\alpha = 0$ .

*Ans.*  $E = 2187$  lbs.

\* EX. 16.  $\epsilon = 33^\circ 42'$ ,  $\phi = 36^\circ$ ,  $H = 3$  ft.,  $\gamma = 120$  lbs.,  $\alpha = 0$ . Determine  $E$ .

*Ans.*  $E = 278$  lbs.

\* EX. 17.  $\phi = 25^\circ$ ,  $\epsilon = 0$ ,  $\alpha = 0$ ,  $H = 4$  ft.,  $\gamma = 120$  lbs.,  $E = ?$

*Ans.*  $E = 390$  lbs.

\* EX. 18.  $\phi = 38^\circ$ ,  $\epsilon = 0$ ,  $\alpha = 0$ ,  $H = 3$  ft.,  $\gamma = 94$  lbs.,  $E = ?$

*Ans.*  $E = 100.5$  lbs.

\* EX. 19. A ditch 6 feet deep is cut with vertical faces in clay. These are shored up with boards, a strut being put across from board to board 2 feet from bottom, at intervals of 5 feet apart. The coefficient of friction of the moist clay is 0.287, and its weight 120 lbs. per cubic foot. Find the thrust on a strut, also find the greatest thrust which might be put upon the struts before the adjoining earth would heave up.

*Ans.*  $E = 1230$  lbs.

Thrust per strut = 6128 lbs.

Greatest thrust = 19029 lbs.

\* Ex. 20. A wall 10 ft. high, 2 ft. thick, and weighing 144 lbs. per cubic ft., is founded in earth weighing 112 lbs. per cubic ft., and whose angle of repose is  $32^\circ$ . Find the least depth of the foundation.

*Ans.*  $x' = 1.21$  ft.  $10 - 1.21 = 8.79$  ft. = amount of wall above the ground.

\* Ex. 21. An iron column is to bear a weight of 20 tons (2240 lbs. = one ton); the foundation is a stone 3 ft. square on bed, sunk in earth weighing 120 lbs. per cu. ft.; angle of repose  $27^\circ$ . Find the least depth to which it must be sunk for equilibrium.

*Ans.*  $x' = 6$  ft.

\* Ex. 22. A brick wall, allowing for openings, weighs 42000 lbs. per rood of 36 sq. ft. (on an average one brick and a half), and stands 45 ft. above the ground; the foundation is to widen to four bricks at the bottom. Find depth of foundation in clay weighing 130 lbs. per cu. ft. (angle of repose  $27^\circ$ ), neglecting weight of unknown foundation.

*Ans.*  $x' = 1.7$  ft.

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\* Alexander's Applied Mechanics.

## PART II.

### THE THEORY OF EARTH-PRESSURE AND THE STABILITY OF RETAINING-WALLS.

*Preliminary Principles.*—Before demonstrating the general formula for the thrust of earth against a wall, it will be necessary to establish the relations between the stresses in an unconfined and homogeneous granular mass.

\* In Fig. 1 let  $ABC$  be any small prism within a granu-

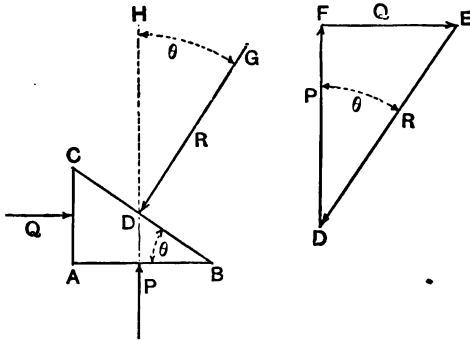


FIG. 1.

lar mass which is in equilibrium under the action of the three stresses  $P$ ,  $Q$ , and  $R$ , having the intensities  $p$ ,  $q$ , and  $r$  respectively.

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\* In all the demonstrations which follow, the dimension perpendicular to the page will be considered as unity.

Let  $\theta$  represent the angle of inclination of the plane  $CB$  with  $AB$ , and the angle at  $A$  be a right angle.

The planes  $AB$  and  $AC$  are called planes of principal stress, and  $P$  and  $Q$  are called principal stresses.

CASE I. *If the principal stresses are of the same kind and their intensities the same, then will the resultant stress on any third plane be normal to that plane and its intensity be equal to that of either principal stress.*

In Fig. 1, for convenience, let  $AB = 1$ , then  $AC = \tan \theta$ , and  $CB = \frac{1}{\cos \theta}$ . Hence

$$P = p, \quad Q = q \tan \theta = p \tan \theta, \text{ since } p = q, \text{ and } R = \frac{r}{\cos \theta}.$$

Since  $P$ ,  $Q$ , and  $R$  are in equilibrium, they will form a closed triangle, as shown on the right in Fig. 1. Hence

$$R^2 = P^2 + Q^2,$$

or

$$\frac{r^2}{\cos^2 \theta} = p^2 + p^2 \tan^2 \theta = p^2(1 + \tan^2 \theta);$$

$$\therefore r = p = q.$$

Also,  $R \cos FDE = P,$

or  $\frac{r}{\cos \theta} \cos FDE = p;$  but  $r = p.$

Hence  $\cos \theta = \cos FDE = \cos HDG;$

$$\therefore HDG = \theta \text{ and } R \text{ is normal to } CB.$$

CASE II. *If the principal stresses are not of the same kind but their intensities the same, then will the resultant make the angle  $\theta$  with the direction of the principal stress, but on the opposite side from that on which the resultant in Case I lies, and its intensity be equal to that of either principal stress.*

The demonstration of Case I proves this principle if Fig. 1 is replaced by Fig. 2.

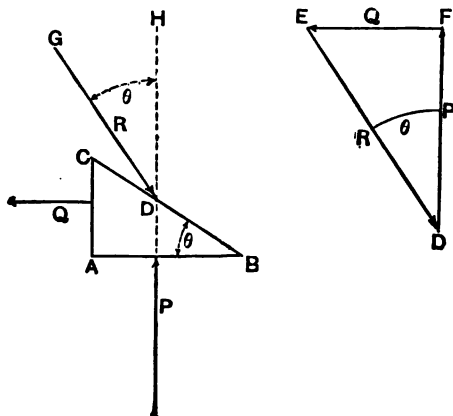


FIG. 2.

CASE III. *Given the principal stresses of the same kind but having unequal intensities, to determine the intensity and direction of the resultant stress on any third plane.*

Let  $P$  and  $Q$  be compressive and the intensity  $p >$  the intensity  $q$ .

The following identities can be written:

$$p = \frac{1}{2}(p + q) + \frac{1}{2}(p - q),$$

and

$$q = \frac{1}{2}(p + q) - \frac{1}{2}(p - q);$$

or the resultant intensity on the plane  $CB$  may be considered as being the resultant of two intensities, one being the intensity of the resultant stress caused by two like principal stresses having the same intensity  $\frac{1}{2}(p + q)$ , and the other the intensity of the resultant stress caused by two unlike principal stresses having the same intensity  $\frac{1}{2}(p - q)$ .

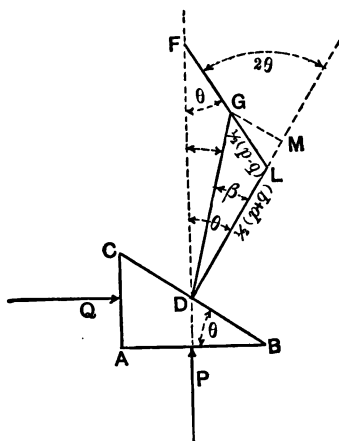


FIG. 3.

The intensity of the resultant stress caused by the first two principal stresses will be, by Case I,  $\frac{1}{2}(p + q)$ , and the direction of the resultant will be normal to the plane  $CB$ . By Case II the resultant of the second pair of principal stresses will make the angle  $\theta$  with the direction of  $P$ , and its intensity will be  $\frac{1}{2}(p - q)$ ; then the resultant intensity can be found as follows:

In Fig. 3 draw  $MD$  normal to  $BC$ , and make  $LD = \frac{1}{2}(p + q)$ ; with  $L$  as a centre and  $LD$  as radius, describe an arc cutting  $FD$  at  $F$ . Then the angle  $LF D = LDF = \theta$ . Lay off  $LG = \frac{1}{2}(p - q)$ , and draw  $GD$ , which is the result-

ant intensity, and the intensity of the resultant stress on  $CD$  caused by the two principal stresses  $P$  and  $Q$ .  $GD$  also represents the direction of the resultant stress  $R$ .

Since the intensities of the principal stresses remain constant,  $\frac{1}{2}(p + q)$  and  $\frac{1}{2}(p - q)$  will remain the same for any inclination of the plane  $CB$ ; hence the intensity  $r$  of the resultant depends upon the angle  $\theta$  when  $p$  and  $q$  are given.

From Fig. 3,

$$GL \cos 2\theta = LM \quad \text{and} \quad GL \sin 2\theta = GM,$$

$$DM = DL + LM = \frac{1}{2}(p + q) + \frac{1}{2}(p - q) \cos 2\theta,$$

$$\overline{GD}^2 = r^2 = \overline{GM}^2 + \overline{DM}^2,$$

or

$$r = \sqrt{p^2 \cos^2 \theta + q^2 \sin^2 \theta}, \quad . \quad . \quad . \quad . \quad (a)$$

which is the general expression for the intensity of the resultant stress of a pair of principal stresses.

As the angle  $\theta$  changes, the angle  $\beta$  will also change, and it will have its maximum value when the angle  $LGD = 90^\circ$ . This is easily proven as follows:

With  $L$  as centre and  $GL$  as radius describe an arc; then  $\beta$  will have its maximum value when the line  $DG$  is tangent to the arc; but when  $DG$  is tangent to the arc the angle  $LGD$  is a right angle, since  $LG$  is the radius of the arc.

$$\sin \max \beta = \frac{p - q}{p + q}, \quad . \quad . \quad . \quad . \quad (b)$$

from which the following can be easily obtained:

$$\frac{p}{q} = \frac{1 - \sin \max \beta}{1 + \sin \max \beta}, \quad . \quad . \quad . \quad . \quad (c)$$

which expresses the limiting ratio of the intensities of the principal stresses consistent with equilibrium,  $p$  being greater than  $q$ .

CASE IV. *Given the intensity and direction of the resultant stress on any plane, and the value of  $\max \beta$ , to determine the intensities and directions of the principal stresses,*

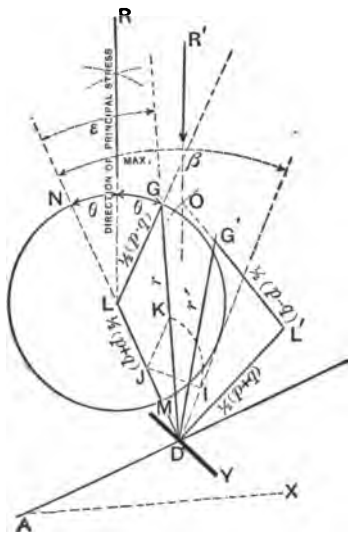


FIG. 4.

Let  $AD$  represent the given plane and  $GD$  the direction and intensity of the resultant stress at the point  $D$ .

Draw  $DL$  normal to  $AD$ , and draw  $DI$ , making the angle  $\max \beta$  with  $LD$ . At any point  $J$  in  $DL$  describe an arc tangent to  $DI$ , cutting  $GD$  in  $K$  and draw  $GL$  parallel to  $KJ$ ; with  $L$  as a centre and  $LG$  as radius describe

a circumference. This circumference will pass through  $G$  and be tangent to  $DI$ ; hence  $\frac{GL}{DL} = \sin \max \beta$ .

Since  $\sin \max \beta = \frac{p-q}{p+q}$ , and  $GL$  and  $LD$  are components of  $r$ ,

$$GL = \frac{1}{2}(p - q) \quad \text{and} \quad DL = \frac{1}{2}(p + q);$$

$$\text{then } ND = NL + LD = \frac{1}{2}(p - q) + \frac{1}{2}(p + q) = p,$$

$$\text{and } MD = LD - LM = \frac{1}{2}(p + q) - \frac{1}{2}(p - q) = q,$$

which completely determines the intensities of the principal stresses.

According to Case III, the direction of the greater principal stress bisects the angle between the prolongation of  $LM$  and the line  $GL$ ; hence  $RL$  represents the direction of the greater principal stress, and that of the other is at right angles to  $RL$ .

The above intensities and directions being determined, the intensity of the resultant stress on any other plane passing through  $D$  is easily determined as follows:

Let  $DY$  represent any plane passing through  $D$ , draw  $DL'$  normal to  $MY$  and equal to  $\frac{1}{2}(p + q)$ . Draw  $R'D$  parallel to  $RL$ , and with  $L'$  as a centre and  $L'D$  as radius describe an arc cutting  $R'D$  at  $O$ , and make  $L'G' = \frac{1}{2}(p - q)$ ; then  $G'D = r' =$  the intensity of the resultant stress on  $DY$ .

It is clear that if the value of  $\max \beta$  can be obtained for a mass of earth that the construction of Fig. 3 can be employed in determining the intensity of the earth-pressure at any point in *any plane* within the mass.

It has been established by experiment that if a body be placed upon a plane, that (as the plane is made to incline to the horizontal) at some angle of inclination the body will commence to slide down the plane, and that this angle depends largely upon the *character* of the surfaces in contact.

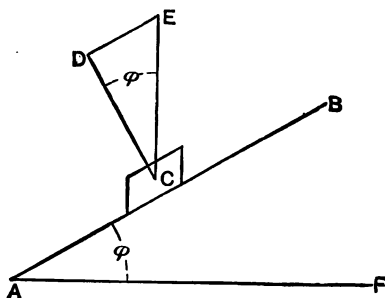


FIG. 5.

In Fig. 5 let  $AB$  represent a plane inclined at the angle  $\phi$  with the horizontal, and  $C$  any mass just on the point of sliding down the plane. Let  $EC$  represent the weight of the mass  $C$ , and  $ED$  and  $DC$  the components respectively parallel and normal to the plane  $AB$ . Then  $DE$  is the force required to just keep the mass  $C$  from sliding down the plane, assuming the plane to be perfectly smooth, or if the plane is rough this force represents the effect of friction.

$$\frac{DE}{DC} = \tan \phi,$$

or when the mass  $C$  is about to slide, the resultant pressure  $EC$  on  $AB$  makes the angle  $\phi$  with the normal to the

plane, the angle  $\phi$  being the inclination of the plane  $AB$ , and is called the angle of friction.

In the case of earth, considered as a dry granular mass, the inclination of the steepest plane upon which earth will not slide is called the angle of repose, and the plane the surface of repose.

From the above, then, it follows that in a mass of earth the resultant pressure on any plane cannot make an angle with the normal to that plane which is greater than the angle of repose  $\phi$ ; therefore the construction of Case IV applies to earth when  $\max \beta$  is replaced by  $\phi$ . The values of  $\phi$  for earth under various conditions are given in Table II.

The preceding principles will now be applied in determining the thrust of earth against a retaining-wall.

### EARTH-PRESSURE.

In order that the formulas may not become too complex for practical use, it will be assumed that the earth is a homogeneous granular mass without cohesion. The surface of the earth will be considered to be a plane, and the length of the mass measured normally to the page as unity.

*\* Given the intensity and direction of the resultant stress at any point in any plane parallel to the surface of the earth, the inclination of the surface of the earth with the horizontal, and the angle of repose, to determine the intensity and direction of the resultant stress on a vertical plane passing through the same point.*

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\* For comparison, see the "Technic," 1888; a construction by Prof. Greene.

The construction follows (see Fig. 4, above) directly from Rankine's Ellipse of Stress.

In Fig. 6 let  $BQ$  represent the surface of the earth, and  $D$  any point in the plane  $AD$  parallel to  $BQ$ ; draw  $DQ$  normal to  $AD$ , and make the vertical  $GD$  equal to  $QD$ ; then  $GD \cdot \gamma$  is the intensity of the resultant pressure at  $D$ . Draw  $DM$ , making the angle  $\phi$  with  $LD$ , and with  $L$  as centre describe an arc tangent to  $DM$  and passing through  $G$ ; then by Case IV  $LG \cdot \gamma = \frac{1}{2}(p - q)$ ,  $LD \cdot \gamma = \frac{1}{2}(p + q)$ ,

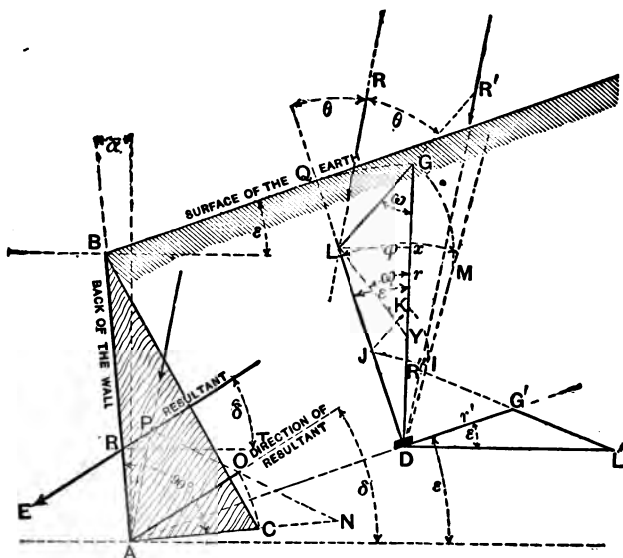


FIG. 6.

and  $RL$  bisecting the angle  $QLG$  is the direction of the greater principal stress. To determine the intensity and direction of the resultant stress at  $D$  on a vertical plane, proceed according to Case IV. Draw  $R'D$  parallel to  $RL$  and  $DL' = DL$  normal to  $DG$ . With  $L'$  as a centre and  $L'D$  as radius describe an arc cutting  $R'D$  at  $R''$ , and make

$L'G' = LG$ ; then  $DG'$  represents the direction of the resultant stress, and  $DG' \cdot \gamma$  the intensity of the resultant.

In Fig. 6 the angle  $R'DL' = DR''L' = 90^\circ - \omega + \theta'$ .  
 $\therefore G'L'D = 2\omega - 2\theta'$ . But  $2\theta' = \omega + \epsilon$ ; hence  $G'L'D = \omega - \epsilon$ .

Draw  $LY = LG$ ; then the angle  $DLY = \omega - \epsilon$ .  $\therefore$  Since  $LD = DL'$  and  $LY = LG = L'G'$ , the triangle  $G'L'D$  equals the triangle  $LYD$  and the angle  $G'DL' = \epsilon$ ; or *the direction of the resultant earth-pressure against a vertical plane is parallel to the surface of the earth.*

From Fig. 6,

$$\frac{1}{2}(p - q) \cos \omega = GX \cdot \gamma,$$

$$\frac{1}{2}(p - q) \sin \omega = LX \cdot \gamma,$$

$$\frac{1}{2}(p + q) \cos \epsilon = DX \cdot \gamma.$$

Now  $DY = DG' = DG - 2GX,$

or

$$\begin{aligned} DG' \cdot \gamma &= DG \cdot \gamma - (p - q) \cos \omega \\ &= \frac{1}{2}(p + q) \cos \epsilon - \frac{1}{2}(p - q) \cos \omega, \end{aligned}$$

$$\frac{1}{2}(p + q) : \sin \omega :: \frac{1}{2}(p - q) : \sin \epsilon,$$

and

$$\sin \omega = \frac{p + q}{p - q} \sin \epsilon,$$

or

$$\cos \omega = \sqrt{1 - \left(\frac{p + q}{p - q}\right)^2 \sin^2 \epsilon} = \sqrt{\frac{(p - q)^2 - (p + q)^2 \sin^2 \epsilon}{(p - q)^2}},$$

and since  $\frac{1}{2}(p + q) \sin \phi = \frac{1}{2}(p - q),$

$$\cos \omega = \frac{1}{\sin \phi} \sqrt{\cos^2 \epsilon - \cos^2 \phi}.$$

Substituting this value for  $\cos \omega$  in the equation for  $DG' \cdot \gamma$ , it becomes

$$DG' \cdot \gamma = \frac{1}{2}(p+q) \cos \epsilon - \frac{1}{2}(p-q) \cdot \frac{1}{\sin \phi} \sqrt{\cos^2 \epsilon - \cos^2 \phi},$$

or since 
$$\frac{1}{\sin \phi} = \frac{p+q}{p-q},$$

$$DG' \cdot \gamma = \frac{1}{2}(p+q) \{ \cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi} \}.$$

In a similar manner,

$$DG \cdot \gamma = \frac{1}{2}(p+q) \{ \cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi} \},$$

and

$$\frac{DG'}{DG} = \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}};$$

hence

$$DG' = DG \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}}.$$

Let  $x$  = the *vertical* distance between the two planes  $BQ$  and  $AD$ , then

$$DG = DQ = x \cos \epsilon.$$

$$\therefore DG' \cdot \gamma = (x) \gamma \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}},$$

which is the expression for the intensity of the resultant earth-pressure on a vertical plane at any depth  $x$  below the surface.

Let

$$* A = \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}}. \quad (d)$$

---

\* See Rankine's Applied Mechanics; Alexander's Applied Mechanics; Theories of Winkler and Mohr.

The average intensity of the resultant earth-pressure on a vertical plane of the length  $x$  will be

$$\left(\frac{x}{2}\right)\gamma A,$$

and hence the total pressure will be

$$P = \frac{x^2}{2} \gamma A. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e)$$

Since the intensities of the pressures are uniformly varying from the surface, and increasing as  $x$  increases, the application of the resultant thrust will be at a depth of  $\frac{2}{3}x$  below the surface.

Considering the earth as an unconfined mass, the above formula is perfectly general and can be applied under all conditions, including the case when  $\epsilon$  is negative.

The resultant stress on any plane as  $AB$ , Fig. 6, can be found by applying the principles of Case IV. Draw  $PA$  parallel to  $RL$ , make  $AN = LD$  and  $NO = LG$ ; then  $AO$  represents the direction of the resultant pressure on  $AB$ . Make  $AC = AO$ ; then the area of the triangle  $ABC$  multiplied by  $\gamma$  is the total pressure on the plane  $AB$ , and this pressure is applied at  $\frac{2}{3}AB$  below  $B$ .

In unconfined earth this construction is perfectly general and applies to *any plane*. It also applies equally well to curved profiles. An example illustrating the application of the method will be given in the *applications*. See pages 22 and 23.

The following graphical construction, Fig. 7, is more convenient than that of Fig. 6.

As before, let  $BE$  represent the surface of the earth, and



proved as follows. The triangle  $GLF$  of Fig. 7 equals the triangle  $GLD$  of Fig. 6.

$$\therefore GL \cdot \gamma = \frac{1}{2}(p - q) \quad \text{and} \quad LF \cdot \gamma = LO \cdot \gamma = \frac{1}{2}(p + q).$$

In Fig. 6, the angle  $NAP = NPA = 90^\circ - \frac{1}{2}(\omega - \epsilon) - \alpha$ .

$$\therefore ONA = \omega - \epsilon + 2\alpha.$$

In Fig. 7, the angle  $OLN = 2\epsilon - 2\alpha$ . But  $GLN = \omega + \epsilon$ .

$$\therefore GLO = \omega - \epsilon + 2\alpha,$$

and  $GO$  of Fig. 7 equals  $AO$  of Fig. 6.

In Fig. 7, the angle  $QNO = 90^\circ - \beta'$ .

In Fig. 6, the angle  $OAB = 90^\circ - \beta'$ .

Therefore the direction of the thrust is the same in both constructions.

The two constructions given above are all that is required to determine the thrust of earth upon any plane within the mass of earth, as one can be used as a check upon the other; but as a formula is often very convenient, a general formula will now be deduced which will enable one to determine the values of  $E$  and  $\delta$  for any plane within a mass of earth.

#### GENERAL FORMULA FOR THE THRUST OF EARTH.

In Fig. 8, let  $BQ$  represent the surface of the earth and  $AB$  any plane upon which the earth-pressure is desired.

Draw  $AD$  parallel to  $BQ$  and let the vertical distance  $QD = FA = x$ ,



Then from Fig. 8,

$$V = \frac{H^2 \gamma}{2} \tan \alpha (1 + \tan \alpha \tan \epsilon) \\ = \frac{H^2 \gamma}{2} \frac{\sin \alpha \cos (\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon}, \quad (g)$$

$$E = \sqrt{(V + P \sin \epsilon)^2 + (P \cos \epsilon)^2} = \sqrt{V^2 + P^2 + 2VP \sin \epsilon}.$$

Substituting (f) and (g) in this it becomes

$$E = \frac{H^2 \gamma}{2} \frac{\cos (\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon} \times \\ \sqrt{\sin^2 \alpha + 2 \sin \alpha \sin \epsilon \cos (\epsilon - \alpha) \frac{A}{\cos \epsilon} + \cos^2 (\epsilon - \alpha) \frac{A^2}{\cos^2 \epsilon}},$$

which becomes, by replacing  $A$  by its value from (d),

$$E = \frac{H^2 \gamma}{2} \frac{\cos (\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon} \times \\ \sqrt{\begin{aligned} &+ \sin^2 \alpha \\ &+ 2 \sin \alpha \sin \epsilon \cos (\epsilon - \alpha) \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \\ &+ \cos^2 (\epsilon - \alpha) \left\{ \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \right\}^2 \end{aligned}}, \quad (1)$$

which is the general equation for the thrust of earth upon any plane within the mass.

To determine the direction of the thrust of the earth, let  $\delta$  be the angle which the direction of the thrust makes with the horizontal; then, from Fig. 8,

$$\tan \delta = \frac{V}{P \cos \epsilon} + \tan \epsilon,$$

Substituting the values of  $V$  and  $P$  given above, this becomes

$$\tan \delta = \frac{\sin \alpha \cos \epsilon + \sin \epsilon \cos (\epsilon - \alpha) A}{\cos \epsilon \cos (\epsilon - \alpha) A}, \quad (1a)$$

where

$$A = \cos \epsilon \frac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}}. \quad (d)$$

Equations (1) and (1a) are readily reduced to more simple forms for special cases. These forms will be found in Part I.

*The Plane of Rupture.*—Although it is not necessary to know the position of the plane of rupture in order to determine the thrust of the earth, yet it may be of interest to know its position, which can be easily determined as follows:

The plane of rupture will be back of the wall and pass through the heel of the wall. The resultant earth-pressure will make the angle  $\phi$  with the normal to this plane. Now the tangent of the angle which the direction of the resultant earth-pressure on any plane makes with the horizontal is determined from the formula

$$\tan \delta = \frac{\sin \alpha}{\cos (\epsilon - \alpha) A} + \tan \epsilon.$$

If  $\omega$  represents the angle which the plane of rupture makes with the vertical passing through the heel of the wall,  $\alpha = \omega$  and  $\delta = \phi + \omega$ .

$$\tan (\phi + \omega) = \frac{\sin \omega}{\cos (\epsilon - \omega) A} + \tan \epsilon,$$

from which the value of  $\omega$  can be determined for any case.

For the case where  $\epsilon = \phi$ ,  $\epsilon$  being positive with respect to the wall and *negative with respect to the plane of rupture*, the above equation becomes

$$\tan (\phi + \omega) = \frac{\sin \omega}{\cos (\phi + \omega) \cos \phi} - \tan \phi,$$

which is satisfied when  $\omega = 90^\circ - \phi$ .

For the case where  $\epsilon = 0$ ,

$$\tan (\phi + \omega) = \frac{\sin \omega}{\cos \omega \tan \left( 45^\circ - \frac{\phi}{2} \right)},$$

which is satisfied when  $\omega = 45^\circ - \frac{\phi}{2}$ .

*Reliability of the Preceding Theory.*—The preceding theory is based upon the assumptions that the earth is a homogeneous mass and without cohesion, and the formulas are deduced under the assumption that the surface of the earth is a plane.

All writers on the subject have considered the earth as a homogeneous mass and, with a few exceptions, without cohesion.

Old and recent experiments indicate that cohesion has very little effect upon the pressure of the earth, which explains why it has not been considered by most writers.

The assumption of a plane earth-surface is necessary whenever practical formulas and direct graphical constructions for obtaining the thrust of the earth are obtained. General formulas can be deduced for any character of surface, but they are too complex for practical use. Those graphical constructions which do not require a plane earth-

surface are not direct in their solution of the problem, but require a series of trials to obtain the maximum thrust.

If the earth-surface is not a plane, one can be assumed which will give the thrust of the earth sufficiently exact for all practical purposes.

For unconfined earth no exceptions can be taken to the preceding theory, the assumptions upon which it is based being accepted, and for confined earth the theory must be true when the direction of the principal stress passing through the heel of the wall lies entirely within the earth.

For all cases in which  $\alpha$  and  $\epsilon$  are positive the theories of *Rankine*, *Winkler*, *Weyrauch*, and *Mohr* agree and give identical results with the preceding theory, as they should, being founded upon the same assumptions.

When  $\alpha$  is negative *Weyrauch* does not consider his theory reliable, and his equations lead to indeterminate results.

*Winkler* and *Mohr* consider their theories reliable whenever the direction of the principal stress passing through the heel of the wall lies entirely within the earth.

*Rankine's* method of considering the case where  $\alpha$  is negative is equivalent to assuming that the introduction of a wall does not affect the stresses within the mass.

It may be concluded that the preceding theory is perfectly exact when  $\alpha$  and  $\epsilon$  are positive; and when  $\alpha$  or  $\epsilon$  is negative that the stresses obtained will be the maximum which under any circumstances can exist.

For the case where  $\epsilon$  is negative the stress obtained (which represents the maximum thrust the wall can have against the earth and have equilibrium) will be considerably larger than the actual stress (when a wall is introduced), depending upon the magnitude of  $\epsilon$ . For small values of  $\epsilon$  the results will be practically correct. For large values of  $\epsilon$

the following method can be employed in determining the thrust of the earth. The method depends upon the *assumption* that the pressure of the earth is normal to the back of the wall. This may or may not be the case, but it appears to be the most consistent assumption to make for this rare and not important case.

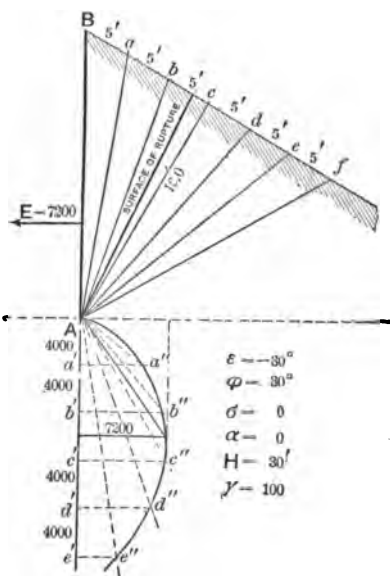


FIG. 8a.

\* In Fig. 8a, let  $AB$  be the back of the wall and  $Bf$  the surface of the earth. Make  $Ba = ab = bc = cd = \text{etc.}$  Some prism  $BAA$  or  $BAb$  or  $BAc$ , etc., will produce the maximum thrust on the wall; and when this maximum thrust is produced, the resultant pressure on the plane  $Aa$

\* See Van Nostrand's Magazine, xvii, 1877, p. 5. "New Constructions in Graphical Statics," by H. T. Eddy, C.E., Ph.D.

or  $Ab$  or  $Ac$ , etc., will make the angle  $\phi$  with the normal to the plane.

On the vertical line  $Ad'$  lay off  $Aa' = a'b' = b'c'$ , etc., and draw  $Aa''$  making the angle  $\phi$  with the normal to  $Aa$ ,  $Ab''$  making the angle  $\phi$  with the normal to  $Ab$ , etc.; then draw  $a'a''$ ,  $b'b''$ , etc., perpendicular to  $AB$ , and draw a curve through  $Aa''$ ,  $b''$ ,  $c''$ , etc. Then there will be a maximum distance parallel to  $a'a''$  between  $Ad'$  and this curve which will be proportional to the thrust of the earth against  $AB$ . This maximum distance multiplied by the altitude  $Ac \div 2$  and the product by  $\gamma$ , the weight of a cubic foot of earth, will be the pressure of the earth.

This method is perfectly general and can be applied in any case.

If the earth-pressure is assumed to have the direction given by the formulas of the preceding theory, the construction will give the same value of  $E$ , the pressure of the earth.

Some writers assume that the direction of  $E$  makes the angle  $\phi'' = \phi$  with the normal to the back of the wall in all cases. This assumption cannot be correct until the wall commences to tip forward, and then it is doubtful that such is the case unless the earth and wall are perfectly dry.

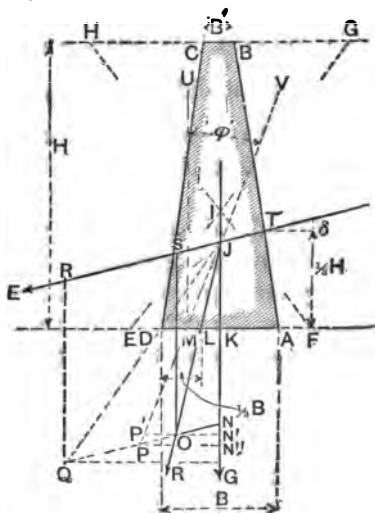
To be on the side of safety in every case, it is better to take the direction of  $E$  as given by the above theory.

The construction of Fig. 8a will give the maximum thrust for any assumed direction for any case.

### TRAPEZOIDAL WALLS.

It will be assumed that the direction and magnitude of the earth-pressure is known, that the position and extent of the back of the wall and the width of the top are given,

to determine the width of the base for stability against overturning, sliding, and crushing of the material.



**FIG. 9.**

*Stability against Overturning.*—Let  $ABCD$ , Fig. 9, represent a section of a trapezoidal wall,  $TR$  the direction of the earth-thrust,  $JG$  the vertical passing through the centre of gravity of the wall, and  $JO$  the direction of the resultant pressure on the base  $AD$  caused by  $E$  and  $G$ .

As long as  $R$  cuts the base  $AD$ , the wall will be stable against overturning. When  $R$  takes the direction  $JQ$ , the wall may be said to be on the point of overturning; then the factor of safety against overturning is  $\frac{QN}{ON}$ , where  $ON$  is the actual value of  $E$ , and  $QN$  the value of  $E$  required to make the resultant  $R$  pass through  $D$ .

*Stability against Sliding.*—Since the wall will not slide

along the surface  $DA$  until the resultant  $R$  makes an angle with the normal to  $DA$  greater than the angle of friction  $\phi'$ , the factor of safety against sliding can be obtained as follows: Draw  $JP$  making the angle  $JMU = \phi'$ ; then the factor of safety against sliding is  $\frac{PN}{ON}$ , where  $PN$  is the force required in the direction of  $E$  to make  $R$  make the angle  $\phi'$  with the normal to  $AD$ , and  $ON$  the actual value of  $E$ .

*Stability against the Crushing of the Material.*—In ordinary practice walls for retaining earth are not of sufficient height to cause very large pressures at their bases, but it is necessary to consider the subject on account of the tendency of the bed-joints to open under certain conditions.

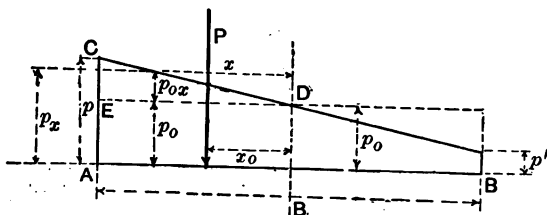


FIG. 10.

Let  $AB$ , Fig. 10, represent any bed-joint in the wall,  $P$  the vertical resultant pressure upon the joint, and  $x_o$  the distance of the point of application from the centre of the joint.

The intensity of  $P$  can be considered as composed of a uniform intensity  $p_o = \frac{P}{B}$ , and a uniformly varying intensity  $p'_o$ , so that  $p_x = p_o + p'_o$ . Let  $a$  equal the tangent of the angle  $CDE$ , then  $p'_o = ax$  and  $p_x = p_o + ax$ .

The pressure upon a surface ( $dx$ )—the joint being considered unity in the dimension normal to the page—is

$$p_x dx = p_0 dx + a x dx,$$

and the moment of this about  $DB$  is

$$(p_0 dx + a x dx)x.$$

The algebraic sum of these moments for values of  $x$  between the limits  $\pm \frac{B}{2}$  must equal  $Px_0$ , or

$$Px_0 = \int_{-\frac{1}{2}B}^{+\frac{1}{2}B} (p_0 x dx + a x^2 dx).$$

Integrating,

$$a = \frac{12x_0 P}{B^3} = \frac{12x_0 p_0}{B^3},$$

and

$$p_x = \frac{B^2 + 12xx_0}{B^3} p_0,$$

or

$$p = \left\{ 1 + \frac{6x_0}{B} \right\} \frac{P}{B};$$

and if  $x_0$  be replaced by  $\frac{1}{2}B - Q$ , where  $Q$  is the distance from  $A$  to the point where  $P$  cuts the base, (Fig. 11,)

$$p = 2 \left( B - \frac{3Q}{B} \right) \frac{P}{B},$$

and

$$p' = 2 \left( 1 - B + \frac{3Q}{B} \right) \frac{P}{B}.$$

If  $Q = \frac{1}{3}B$ ,

$$p' = 0 \quad \text{and} \quad p = 2p_0.$$

from which it is seen that when  $R$  cuts the base outside the middle third, the joint will have a tendency to open at points which are at a maximum distance from  $R$  where it cuts the base.

Therefore in no case should the resultant pressure be permitted to cut the base outside the middle third. This makes it unnecessary to consider the stability against overturning.

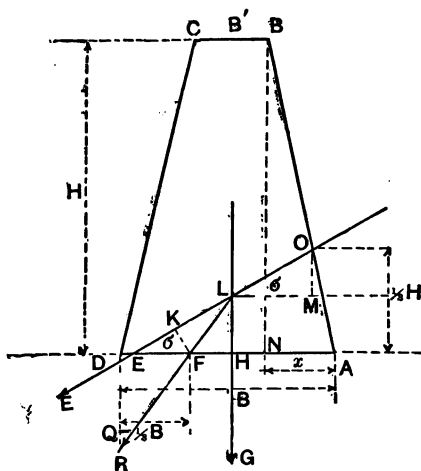


FIG. 11.

Then in designing a wall the following conditions must exist for stability:

I. *The resultant  $R$  must cut the base for stability against overturning.*

II. *The resultant  $R$  must not make an angle with the normal to the base of the wall greater than the angle of friction  $\phi'$ .*

III. *The resultant  $R$  must not cut the base outside of the middle third, in order that there may be no tendency for the bed-joints to open.*

The above three conditions apply to any bed-joint of the wall; but if they are satisfied at the base and the wall has the section shown in Fig. 11, it will not be necessary to consider any joints above the base unless the character of the stone or the bonding is different.

*Determination of the width of the base of a retaining-wall under the condition that  $R$  cuts the base at a point  $\frac{1}{3}B$  from the toe of the wall.*

Let  $H$ ,  $B'$ ,  $x$ ,  $\delta$ , and  $E$  be given to determine  $B$ .

From Fig. 11,

$$KF = \frac{x}{3} \sin \delta + \frac{H}{3} \cos \delta - \frac{2B}{3} \sin \delta,$$

$$HD = \frac{2B^2 + 2BB' - Bx - 2B'x - B'^2}{3(B + B')},$$

$$HF = HD - \frac{B}{3} = \frac{B^2 + BB' - Bx - 2B'x - B'^2}{3(B + B')}.$$

For equilibrium

$$E(KF) = G(HF) = \frac{B + B'}{2} HW(HF).$$

Substituting the values of  $KF$  and  $HF$  in the above and reducing, it becomes

$$\begin{aligned} B^2 + B \left( \frac{4E}{HW} \sin \delta + B' - x \right) \\ = \frac{2E}{HW} (H \cos \delta + x \sin \delta) + 2B'x + B'^2, \quad (8) \end{aligned}$$

which is the general equation for the width of the base of a trapezoidal wall.

For a rectangular wall  $B' = B$ .

For a triangular wall  $B' = 0$ .

For a wall with a vertical front  $B' + x = B$  or  $B' = B - x$ .

For a wall with a vertical back  $x = 0$ .

Equation (8) is easily transformed to satisfy the requirements of special cases.

The width of the base can be found graphically by assuming a value for  $B$  and finding the value of  $Q$ ; if it is less than  $\frac{1}{3}B$  another value of  $B$  must be assumed, and so on until  $Q$  is equal to or greater than  $\frac{1}{3}B$ .

*Depth of Foundations.*—Given the angle of repose  $\phi$  of any earth, to determine the depth to which it is necessary to sink a foundation to support a given load. The surface of the earth is assumed to be horizontal.

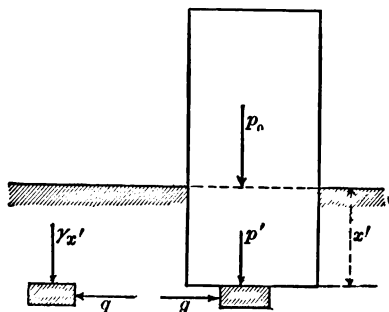


FIG. 12.

**CASE I.** *When the intensity of the pressure on the base of the foundation is uniform.*

In Fig. 12, let  $p_0$  represent the intensity of the pressure on the base of the foundation,

Now when the masonry is about to sink (see Eq. (c)),

$$\frac{p_0}{q} = \frac{1 + \sin \phi}{1 - \sin \phi} \quad \text{or} \quad q = p_0 \frac{1 - \sin \phi}{1 + \sin \phi}.$$

If  $x'$  represents the depth to which the foundation extends below the surface of the earth and  $\gamma$  the weight of a cubic foot of earth, then  $\gamma x'$  equals the vertical intensity of the earth-pressure on a plane at the depth of the lowest point of the foundation.

When the wall is on the point of sinking, the earth must be on the point of rising, or

$$\frac{q}{\gamma x'} = \frac{1 + \sin \phi}{1 - \sin \phi},$$

or

$$p_0 = \gamma x' \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2 \dots \dots \dots (15)$$

In any case  $p_0$  must not have a greater value than that obtained from (15)—

$$x' = \frac{p_0}{\gamma} \left\{ \frac{1 - \sin \phi}{1 + \sin \phi} \right\}^2 = \frac{p_0}{\gamma} \tan^2 \left( 45^\circ - \frac{\phi}{2} \right). \quad (16)$$

The value of  $x'$  as obtained from (16) is the least allowable value consistent with equilibrium. Since  $x'$  is a function of  $\tan^2 \left( 45^\circ - \frac{\phi}{2} \right)$ , care must be taken that  $\phi$  is assumed at its least value. As  $\phi$  becomes smaller the value of  $x'$  increases rapidly.

CASE II. *When the intensity of the pressure on the base is uniformly varying.*

Let  $p$  represent the maximum intensity of the pressure on the earth and  $p'$  the minimum intensity; then for

equilibrium  $p$  must not exceed the value obtained from the following equation:

$$p = x' \gamma \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2 \cdot \cdot \cdot \cdot (17)$$

Also,  $p'$  must never be less than  $x' \gamma$ ; then

$$p_0 = \frac{p + p'}{2} = \frac{x' \gamma}{2} \left\{ 1 + \left( \frac{1 + \sin \phi}{1 - \sin \phi} \right)^2 \right\} = x' \gamma \frac{1 + \sin^2 \phi}{(1 - \sin \phi)^2} \quad (18)$$

which expresses the maximum value which  $p_0$  can have for the equilibrium of the earth. Solving (18) for  $x'$ ,

$$x' = \frac{p_0}{\gamma} \frac{(1 - \sin \phi)^2}{1 + \sin^2 \phi}, \quad \cdot \cdot \cdot \cdot (19)$$

which is the minimum value  $x'$  can have for the equilibrium of the earth.

In order that  $p'$  may never be less than  $x' \gamma$  the resultant pressure on the base of the foundation must cut the base within a certain distance of the centre of the base. If  $x_0$  equal this distance, then (see page 51)

$$p' = \left( 1 - \frac{6x_0}{B} \right) p_0 = x' \gamma.$$

Substituting the value of  $p_0$  from (18) and solving for  $x_0$ ,

$$x_0 = \frac{1}{3} \frac{\sin \phi}{1 + \sin^2 \phi} \cdot \cdot \cdot \cdot (20)$$

which is the maximum value  $x_0$  can have, consistent with the stability of the earth.

*Abutting Power of Earth.*—Let the surface of the earth be horizontal and the body pushing the earth have a verti-

cal face; then at the depth  $x'$  the maximum horizontal pressure per unit of area is (see Case I above)

$$q = x' \gamma \frac{1 + \sin \phi}{1 - \sin \phi},$$

and since  $q$  varies directly as  $x'$ , the total thrust  $P$  which the earth is capable of resisting is

$$P = \frac{(x')^2 \gamma}{2} \frac{1 + \sin \phi}{1 - \sin \phi} \quad . \quad . \quad . \quad (21)$$



# APPENDIX.

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## WEYRAUCH'S THEORY OF THE RETAINING-WALL.\*

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IN the following the earth is supposed without cohesion, and its pressure is determined independently of any arbitrary assumptions as to direction of the earth-pressure, and with sole reference to the three necessary conditions of equilibrium. The single and only supposition, then, is as follows: *That the forces upon any imaginary plane-section through the mass of earth have the same direction.*

This assumption lies at the foundation of *all* theories of earth-pressure against retaining-walls. For those cases, therefore, to which the following discussion does not apply no complete or satisfactory theory is yet possible. In what follows, the ordinary assumption as to the direction of the earth-pressure will be proved to be *incorrect*, except for special cases.

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\* *Zeitschrift für Baukunde*, Band I. Heft 2, 1878.

## I.

## GENERAL RELATIONS.

Let the surface of the earth have any form, and the wall  $AB$ , Fig. 1, have any inclination. The earth-pressure makes any angle,  $\delta$ , with the normal to the wall.

Suppose through the point  $A$  the plane  $AC$ . Then the weight  $G$  of the prism  $ABC$  is held in equilibrium by the

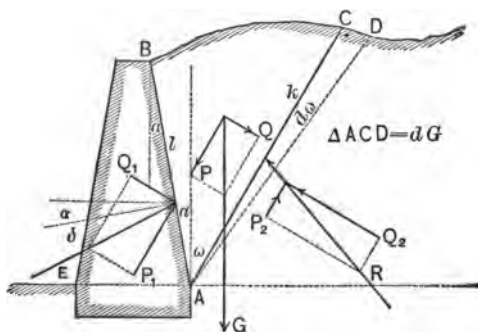


FIG. 1.

reaction of the wall,  $E$ , and by the resultant,  $R$ , of all the forces acting upon  $AC$ .

Now decompose  $E$ ,  $G$ , and  $R$  into components parallel and normal to  $AC$ ; then for every unit in length of the wall, denoting by  $e$ ,  $g$ , and  $r$  the lever-arms of  $E$ ,  $G$ , and  $R$  respectively with reference to  $A$ , the sum of the forces parallel to  $AC = 0$ , or

$$P - P_1 - P_2 = 0; \dots \dots \dots (1)$$

the sum of the forces perpendicular to  $AC = 0$ , or

$$Q + Q_1 - Q_2 = 0; \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the sum of moments about  $A = 0$ , or

$$Gg + Ee - Rr = 0. \quad . \quad . \quad . \quad . \quad (3)$$

Equation (3) was first introduced by Prof. Weyrauch.

Further, according to the theory of friction, if  $\varphi$  is the coefficient of friction for earth on earth,

$$\frac{P_2}{Q_2} \leq \tan \varphi \text{ or } \frac{P - P_1}{Q + Q_1} \leq \tan \varphi. \quad . \quad . \quad (4)$$

If now there is any plane for which

$$P - P_1 = (Q + Q_1) \tan \varphi, \quad . \quad . \quad . \quad (5)$$

this plane  $AC$  will be a plane of equilibrium, and  $\frac{P - P_1}{Q + Q_1}$  will be a maximum, or

$$\frac{d\left(\frac{P - P_1}{Q + Q_1}\right)}{d\omega} = 0. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This plane is designated as the "surface of rupture."

From Fig. 1, for every position of  $AC$ ,

$$\begin{aligned} P &= G \cos \omega, & Q &= G \sin \omega, \\ P_1 &= E \sin (\omega + \alpha + \delta), & Q_1 &= E \cos (\omega + \alpha + \delta). \end{aligned}$$

Substituting the above values of  $P$ ,  $P_1$ ,  $Q$ , and  $Q_1$  in equation (5), it becomes

$$\begin{aligned} G \cos \omega - E \sin (\omega + \alpha + \delta) \\ = [G \sin \omega + E \cos (\omega + \alpha + \delta)] \tan \varphi; \end{aligned}$$

and when  $\omega$  refers to the surface of rupture, the earth-pressure upon  $AB$  becomes

$$E = \frac{\cos \omega - \sin \omega \tan \varphi}{\sin (\omega + \alpha + \delta) + \cos (\omega + \alpha + \delta) \tan \varphi} G.$$

Substituting the value of  $\tan \varphi$  or  $\frac{\sin \varphi}{\cos \varphi}$ , this becomes

$$E = \frac{\cos \varphi \cos \omega - \sin \omega \sin \varphi}{\sin (\omega + \alpha + \delta) \cos \varphi + \cos (\omega + \alpha + \delta) \sin \varphi} G,$$

which becomes

$$E = \frac{\cos (\varphi + \omega)}{\sin (\varphi + \omega + \alpha + \delta)} G. \quad . \quad . \quad (7)$$

In order to refer to the surface of rupture, the following relation must exist :

$$\frac{d \left( \frac{G \cos \omega - E \sin (\omega + \alpha + \delta)}{G \sin \omega + E \cos (\omega + \alpha + \delta)} \right)}{d\omega} = 0. \quad (7a)$$

Performing the differentiation indicated in the equation (7a), considering  $G$  and  $\omega$  as the variables, it becomes

$$\frac{+ [dG \cos \omega - \sin \omega d\omega G - E \cos (\omega + \alpha + \delta) d\omega] [G \sin \omega + E \cos (\omega + \alpha + \delta)] - [dG \sin \omega + \cos \omega d\omega G - E \sin (\omega + \alpha + \delta) d\omega] [G \cos \omega - E \sin (\omega + \alpha + \delta)]}{[G \sin \omega + E \cos (\omega + \alpha + \delta)]^2 d\omega} = 0; \quad . \quad . \quad . \quad (7b)$$

dividing by  $d\omega$ , this becomes

$$\frac{+ \left[ \frac{dG \cos \omega}{d\omega} - [G \sin \omega + E \cos (\omega + \alpha + \delta)] \right] [G \sin \omega + E \cos (\omega + \alpha + \delta)] - \left[ \frac{dG \sin \omega}{d\omega} + [G \cos \omega - E \sin (\omega + \alpha + \delta)] \right] [G \cos \omega - E \sin (\omega + \alpha + \delta)]}{[G \sin \omega + E \cos (\omega + \alpha + \delta)]^2} = 0, \quad . \quad . \quad . \quad (7c)$$

or

$$\begin{aligned} & + \frac{dG \cos \omega}{d\omega} [G \sin \omega + E \cos (\omega + \alpha + \delta)] - [G \sin \omega + E \cos (\omega + \alpha + \delta)]^2 \\ & - \frac{dG \sin \omega}{d\omega} [G \cos \omega - E \sin (\omega + \alpha + \delta)] - [G \cos \omega - E \sin (\omega + \alpha + \delta)]^2 \\ & \frac{[G \sin \omega + E \cos (\omega + \alpha + \delta)]^2}{[G \sin \omega + E \cos (\omega + \alpha + \delta)]^2} = \\ & = 0. \dots \dots \dots (7d) \end{aligned}$$

Now, since

$$\begin{aligned} \cos \omega \cos (\omega + \alpha + \delta) + \sin \omega \sin (\omega + \alpha + \delta) &= \cos (\alpha + \delta) \\ \text{and} \quad \sin^2 \omega + \cos^2 \omega &= 1, \end{aligned}$$

by clearing of fractions this becomes

$$- \frac{EdG \cos (\alpha + \delta)}{d\omega} + G^2 - 2GE \sin (\alpha + \delta) + E^2 = 0. \quad (7e)$$

Now since  $dG = \frac{1}{2}k \cdot d\omega \cdot k\gamma$ , equation (7e) reduces to

$$G^2 - 2GE \sin (\alpha + \delta) - \frac{Ek^2\gamma \cos (\alpha + \delta)}{2} + E^2 = 0, \quad (7f)$$

which becomes, after dividing by  $GE$ ,

$$\frac{G}{E} - 2 \sin (\alpha + \delta) - \frac{k^2\gamma \cos (\alpha + \delta)}{2G} + \frac{E}{G} = 0. \quad (8)$$

Substituting the value of  $\frac{E}{G}$  from equation (7), transposing and multiplying by two, equation (8) reduces to

$$\frac{2 \sin (\phi + \alpha + \omega + \delta)}{\cos (\phi + \omega)} - 4 \sin (\alpha + \delta) + \frac{2 \cos (\phi + \omega)}{\sin (\phi + \omega + \alpha + \delta)} = \frac{k^2\gamma \cos (\alpha + \delta)}{G}, \quad (8a)$$

whence

$$G = \frac{k^2 \gamma \cos(\alpha + \delta)}{\frac{2 \sin(\phi + \omega + \alpha + \delta)}{\cos(\phi + \omega)} - 4 \sin(\alpha + \delta) + \frac{2 \cos(\phi + \omega)}{\sin(\phi + \omega + \alpha + \delta)}}, \quad \dots \dots (8b)$$

which reduces to

$$G = \frac{\cos(\phi + \omega) \sin(\phi + \omega + \alpha + \delta) \cos(\alpha + \delta) k^2 \gamma}{2 [\sin^2(\phi + \omega + \alpha + \delta) - 2 \sin(\alpha + \delta) \cos(\phi + \omega) \sin(\phi + \omega + \alpha + \delta) + \cos^2(\phi + \omega)]}. \quad (8c)$$

Since

$$\begin{aligned} \sin(\phi + \omega + \alpha + \delta) &= \sin(\phi + \omega) \cos(\alpha + \delta) \\ &\quad + \cos(\phi + \omega) \sin(\alpha + \delta), \end{aligned}$$

the parenthetical portion of the denominator becomes

$$\begin{aligned} &\sin^2(\phi + \omega) \cos^2(\alpha + \delta) \\ &\quad + 2 \sin(\alpha + \delta) \cos(\phi + \omega) \sin(\phi + \omega) \cos(\alpha + \delta) \\ &\quad + \cos^2(\phi + \omega) \sin^2(\alpha + \delta) \\ &\quad - 2 \sin(\alpha + \delta) \cos(\phi + \omega) \sin(\phi + \omega) \cos(\alpha + \delta) \\ &\quad - 2 \sin(\alpha + \delta) \cos(\phi + \omega) \cos(\phi + \omega) \sin(\alpha + \delta) \\ &\quad + \cos^2(\phi + \omega), \end{aligned}$$

or

$$\begin{aligned} &\sin^2(\phi + \omega) \cos^2(\alpha + \delta) \\ &\quad - 2 \sin^2(\alpha + \delta) \cos^2(\phi + \omega) \\ &\quad + \sin^2(\alpha + \delta) \cos^2(\phi + \omega) + \cos^2(\phi + \omega), \end{aligned}$$

$$\text{or} \quad \sin^2(\phi + \omega) \cos^2(\alpha + \delta) - \cos^2(\phi + \omega) \sin^2(\alpha + \delta) + \cos^2(\phi + \omega),$$

$$\text{or} \quad \sin^2(\phi + \omega) \cos^2(\alpha + \delta) + \cos^2(\phi + \omega) [1 - \sin^2(\alpha + \delta)],$$

$$\text{or} \quad \sin^2(\phi + \omega) \cos^2(\alpha + \delta) + \cos^2(\phi + \omega) \cos^2(\alpha + \delta),$$

$$\text{or} \quad \cos^2(\alpha + \delta) [\sin^2(\phi + \omega) + \cos^2(\phi + \omega)],$$

which equals  $\cos^2 (\alpha + \delta)$ , and equation (8c) becomes, after dividing by  $\cos (\alpha + \delta)$  and factoring,

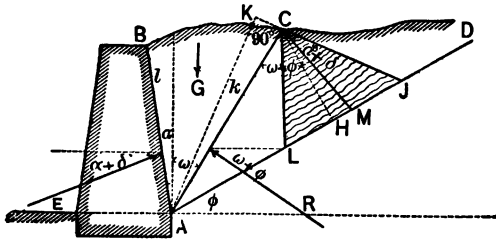
$$G = \frac{\cos(\varphi + \omega) \sin(\varphi + \omega + \alpha + \delta)}{\cos(\alpha + \delta)} \cdot \frac{k^* \gamma}{z} = \text{Function } \gamma, \quad (9)$$

from which

$$\sin (\varphi+\omega+\alpha+\delta)=\frac{2 G}{k^2 \gamma} \cdot \frac{\cos (\alpha+\delta)}{\cos (\varphi+\omega)},$$

which being substituted in equation (7) gives

$$E = \frac{G \cos(\varphi + \omega)}{2G \cos(\alpha + \delta)} = \frac{\cos^2(\varphi + \omega)}{\cos(\alpha + \delta)} \cdot \frac{k^2 \gamma}{2}. \quad (10)$$



**FIG. 2.**

And, since the sum of the horizontal components of  $E$ ,  $G$ , and  $R$  must be equal to 0, or Fig. 2,

$$E \cos (\alpha+\delta)=R \cos (\omega+\varphi),$$

and.

$$R = E \frac{\cos(\alpha + \delta)}{\cos(\omega + \varphi)};$$



which reduces to

$$AJ = \frac{\sin(\varphi + \omega + \alpha + \delta)}{\cos(\alpha + \delta)} k;$$

and hence, according to equation (9),

$$G = \text{Func. } \gamma = \gamma \Delta ACJ. \quad . \quad . \quad . \quad (12)$$

Also, if  $AK$  is perpendicular to  $CJ$ ,

$$\frac{CH}{AK} = \frac{k \cos(\varphi + \omega)}{k \sin(\varphi + \omega + \alpha + \delta)} = \frac{E}{G};$$

and if  $JL$  is made equal to  $JC$ , then, since the perpendicular from  $L$  upon  $CJ$  is equal to  $CH$ ,

$$\frac{\Delta CJL}{\Delta CJA} = \frac{CH}{AK} = \frac{E}{G},$$

$$\text{or} \quad E = \gamma \Delta CJL. \quad . \quad . \quad . \quad (13)$$

If, finally,  $AM = AC$ ,

$$\Delta ACM = \frac{AM \cdot CH}{2} = \frac{1}{2} k^2 \cos(\varphi + \omega),$$

$$\text{or} \quad R = \gamma \Delta ACM. \quad . \quad . \quad . \quad (14)$$

All these geometrical results may be summed up as follows :

Draw from the highest point  $C$  of the surface of rupture a line  $CJ$ , which makes with the normal  $CH$  to the natural slope the angle  $\alpha + \delta$ , or the angle which the earth-pressure makes with the horizontal ; then the  $\Delta ACJ$  is

equal in area to the  $\triangle ABC$ , the prism of rupture. Then lay off  $JL = JC$  and  $AM = AC$  and draw  $CL$  and  $CM$ ; then for every unit in length of the wall the following relations exist :

$$\left. \begin{array}{l} \text{Weight of prism of rupture,} \quad G = \gamma \triangle CAJ; \\ \text{Earth-pressure upon wall,} \quad E = \gamma \triangle CJL; \\ \text{Reaction of the surface of rupture, } R = \gamma \triangle CAM. \end{array} \right\} (14a)$$

The first two relations were first made known by Rebhahn in 1871, for  $\delta = 0$  or  $\varphi$ .

$$\text{Since, now, } G : E : R = AJ : JC : CA, \quad . \quad . \quad . \quad (15)$$

it can be asserted that—

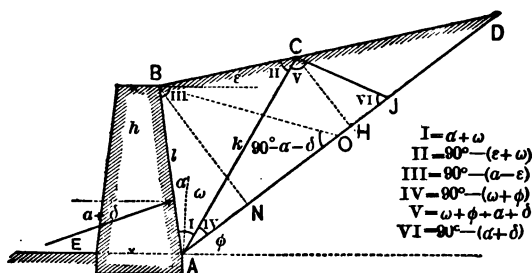
The weight of the prism of rupture and the reactions of the wall and of the surface of rupture are to each other as the three sides of the  $\triangle ACJ$ .

Thus far no assumption whatever has been made as to the value of the angle  $\delta$ . This is determined by equation (3), which, in all theories following Coulomb's method, does not occur.

## II.

## PLANE EARTH-SURFACE INCLINED

ADOPT in this case the notation of Fig. 3, and let  $E$  be first determined for any value of  $\delta$ .



**FIG. 8.**

If  $AC$  is the surface of rupture, then  $\angle ABC = \angle ACJ$ ;  
or, since

$$\frac{AB}{AC} = \frac{\sin II}{\sin III}, \quad AB = AC \frac{\sin II}{\sin III}.$$

In like manner,  $AJ = AC \frac{\sin V}{\sin VI}.$

But since  $\triangle ABC = \triangle ACJ$ ,

$$AB \cdot AC \sin I = AJ \cdot AC \sin IV; \quad (16)$$

$$\text{or} \quad \frac{\sin I \sin II}{\sin III} = \frac{\sin IV \sin V}{\sin VI}; \quad . \quad . \quad . \quad (16a)$$



then determine the point  $J$  so that equation (17) is fulfilled, that is, make  $AJ$  a mean proportional between  $AO$  and  $AD$ ; then draw  $JC$  parallel to  $OB$ . Thus the surface of rupture  $AC$  is found, and use can now be made of the relations already deduced in I.

In order to determine  $J$  ( $A$ ,  $O$ , and  $D$  being given), there are several methods, one of which is indicated in the figure. In all these constructions  $\delta$  is assumed.

Now from equation (13),  $E = \frac{1}{2} \gamma \overline{JC}^2 \cos(\alpha + \delta)$ ,

but

$$\frac{CJ}{BO} = \frac{AD - AJ}{AD - AO} = \frac{AD - \sqrt{AD \cdot AO}}{AD - AO} = \frac{1 - \sqrt{\frac{AO}{AD}}}{1 - \frac{AO}{AD}}.$$

Let  $n = \sqrt{\frac{AO}{AD}}$ , then  $CJ = \frac{1-n}{1-n^2} BO = \frac{BO}{1+n}$ .

From Fig. 3,

$$\frac{AO}{AB} = \frac{\sin(\varphi + \delta)}{\cos(\alpha + \delta)}, \quad \frac{AB}{AD} = \frac{\sin(\varphi - \epsilon)}{\cos(\alpha - \epsilon)};$$

and the multiplication of these equations gives

$$n = \sqrt{\frac{\sin(\varphi + \delta) \sin(\varphi - \epsilon)}{\cos(\alpha + \delta) \cos(\alpha - \epsilon)}}. \quad \dots (18)$$

If  $AB = l$ ,  $BO = \frac{\cos(\varphi - \alpha)}{\cos(\alpha + \delta)} l$ ;

and by substitution of  $BO$  and  $n$  in the value for  $CJ$ , and of  $CJ$  in that for  $E$ ,

$$E = \left[ \frac{\cos(\phi - \alpha)}{n + 1} \right]^2 \frac{l^2 \gamma}{2 \cos(\alpha + \delta)} = \left[ \frac{\cos(\phi - \alpha)}{(n + 1) \cos \alpha} \right]^2 \frac{h^2 \gamma}{2 \cos(\alpha + \delta)}. \quad (19)$$

For the special case of the earth-surface parallel to the angle of repose,  $\varepsilon = \varphi$ ,  $n = 0$ , and

$$E = \frac{\cos^2(\varphi - \alpha)}{\cos(\alpha + \delta)} \frac{l^2 \gamma}{2} = \left[ \frac{\cos(\varphi - \alpha)}{\cos \alpha} \right]^2 \frac{h^2 \gamma}{2 \cos(\alpha + \delta)}. \quad (20)$$

These formulæ hold good for any value of  $\delta$ . But the angle  $\delta$  is determined by equation (3). In order to insert  $e$  and  $r$  in this formula, the points of application of  $E$  and  $R$  must be known. The angles  $\delta$  and  $\omega$  are connected by the relations in (16b), in which there are no other unknown quantities. Since now  $\delta$ , according to the single assumption of Prof. Weyrauch's theory, is independent of the height, so also is  $\omega$ , and then for variable  $h$  equations (19) and (11) become

$$\begin{aligned} E &= Cl^2, & R &= C_1 k^2, \\ dE &= 2Cl dl, & dR &= 2C_1 k dk. \end{aligned}$$

Let  $x$  and  $z$  equal the distance of the point of application of  $E$  and  $R$  from  $A$ , respectively. Now considering the top as the origin or centre of moments,

$$E(l - x) = 2C \int_0^l l^2 dl, \quad R(k - z) = 2C_1 \int_0^k k^2 dk,$$

and therefore  $x = \frac{1}{3}l$  and  $z = \frac{1}{3}k$ .

Now  $G$  must act through the centre of gravity of the  $\triangle ABC$ , and it has been already proved that the points

of application of  $E$  and  $R$  are at distances  $\frac{1}{8}l$  and  $\frac{1}{8}k$  respectively above  $A$ ; hence (Fig. 3')  $ah = ed$  and  $hf = g = bd - ah = \frac{1}{8}k \sin \omega - \frac{1}{8}l \sin \alpha$ .

Substituting these values in equation (3) and referring to equation (15),

$$AB (CJ \cos \delta - AJ \sin \alpha) = AC (AC \cos \phi - AJ \sin \omega), \quad . . . \quad (22)$$

or

$$\sin II (\sin IV \cos \delta - \sin V \sin \alpha) = \sin III (\sin VI \cos \phi - \sin V \sin \omega), \quad (22a)$$

$$\begin{aligned} \text{or} \quad & \cos (\epsilon + \omega) [\cos (\phi + \omega) \cos \delta - \sin (\phi + \omega + \alpha + \delta) \sin \alpha] \\ & = \cos (\alpha - \epsilon) [\cos (\alpha + \delta) \cos \phi - \sin (\phi + \omega + \alpha + \delta) \sin \omega]. \quad . . \quad (22b) \end{aligned}$$

By means of the two equations (16*b*) and (22*b*) the two unknown quantities  $\delta$  and  $\omega$  are completely determined. As soon as these are known,  $E$  can be found from equation (19) or (20). Also by the relations in equations (16) and (22), or (16*a*) and (22*b*), the surface of rupture and direction of the earth-pressure may be determined, and can therefore be found by a graphical construction.

## III.

## HORIZONTAL EARTH-SURFACE.

For this most important practical case it is simply necessary to make  $\varepsilon = 0$  in equation (19). The proper values of  $\delta$  and  $\omega$  in this case are found from (16*b*) and (22*b*).

Making  $\varepsilon = 0$  in equation (22*b*), it becomes

$$\cos \omega [\cos (\varphi + \omega) \cos \delta - \sin (\varphi + \omega + \alpha + \delta) \sin \alpha] \\ - \cos \alpha [\cos (\alpha + \delta) \cos \varphi - \sin (\varphi + \omega + \alpha + \delta) \sin \omega] = 0.$$

Since

$$\sin (\varphi + \omega + \alpha + \delta) = \sin (\varphi + \omega) \cos (\alpha + \delta) \\ + \cos (\varphi + \omega) \sin (\alpha + \delta),$$

$$\cos (\alpha + \delta) = \cos \alpha \cos \delta - \sin \alpha \sin \delta,$$

$$\text{and} \quad \sin (\alpha + \delta) = \sin \alpha \cos \delta + \cos \alpha \sin \delta,$$

the above expression becomes

$$\left. \begin{aligned} & \cos \omega \cos \delta \cos (\varphi + \omega) \\ & - \cos \omega \sin \alpha \cos \alpha \cos \delta \sin (\varphi + \omega) \\ & \quad + \cos \omega \sin^2 \alpha \sin \delta \sin (\varphi + \omega) \\ & - \cos \omega \sin \alpha \cos \alpha \sin \delta \cos (\varphi + \omega) \\ & \quad - \cos \omega \sin^2 \alpha \cos \delta \cos (\varphi + \omega) \\ & - \cos \alpha \cos \varphi \cos (\alpha + \delta) \\ & + \cos^2 \alpha \sin \omega \cos \delta \sin (\varphi + \omega) \\ & \quad - \cos \alpha \sin \omega \sin \alpha \sin \delta \sin (\varphi + \omega) \\ & + \cos^2 \alpha \sin \omega \sin \delta \cos (\varphi + \omega) \\ & \quad + \cos \alpha \sin \omega \sin \alpha \cos \delta \cos (\varphi + \omega) \end{aligned} \right\} = 0,$$

which reduces to

$$\left. \begin{aligned} & \cos \omega \cos (\varphi + \omega) \cos \delta \\ & - \sin \alpha \cos \alpha [\sin (\varphi + \omega) \cos \omega - \cos (\varphi + \omega) \sin \omega] \cos \delta \\ & - \sin \alpha \cos \alpha [\cos (\varphi + \omega) \cos \omega + \sin (\varphi + \omega) \sin \omega] \sin \delta \\ & + [\sin^2 \alpha \sin (\varphi + \omega) \cos \omega + \cos^2 \alpha \cos (\varphi + \omega) \sin \omega] \sin \delta \\ & + [\cos^2 \alpha \sin (\varphi + \omega) \sin \omega - \sin^2 \alpha \cos (\varphi + \omega) \cos \omega] \cos \delta \\ & - \cos^2 \alpha \cos \varphi \cos \delta + \sin \alpha \cos \alpha \cos \varphi \sin \delta \end{aligned} \right\} = 0. \quad (22c)$$

The expression in the first parenthesis is equal to  $\sin \varphi$ , in the second to  $\cos \varphi$ . If in the third  $\cos^2 \alpha = 1 - \sin^2 \alpha$ , and in the fourth  $\sin^2 \alpha = 1 - \cos^2 \alpha$ , equation (22c) becomes

$$\left. \begin{aligned} & + \cos \omega \cos (\varphi + \omega) \cos \delta - \sin \alpha \cos \alpha \cos \delta \sin \varphi \\ & \qquad \qquad \qquad - \sin \alpha \cos \alpha \sin \delta \cos \varphi \\ & + \sin \delta \sin^2 \alpha \sin (\varphi + \omega) \cos \omega + \sin \delta \sin \omega \cos (\varphi + \omega) \\ & \qquad \qquad \qquad - \sin^2 \alpha \sin \omega \sin \delta \cos (\varphi + \omega) \\ & + \cos \delta \cos^2 \alpha \sin (\varphi + \omega) \sin \omega - \cos \delta \cos \omega \cos (\varphi + \omega) \\ & \qquad \qquad \qquad + \cos^2 \alpha \cos \delta \cos \omega \cos (\varphi + \omega) \\ & - \cos^2 \alpha \cos \varphi \cos \delta + \sin \alpha \cos \alpha \cos \varphi \sin \delta \end{aligned} \right\} = 0.$$

Reducing and dividing by  $\cos \delta$ ,

$$\left. \begin{aligned} & - \sin \alpha \cos \alpha \sin \varphi + \sin^2 \alpha \cos \omega \sin (\varphi + \omega) \tan \delta \\ & \qquad \qquad \qquad + \sin \omega \cos (\varphi + \omega) \tan \delta \\ & - \sin^2 \alpha \sin \omega \cos (\varphi + \omega) \tan \delta \\ & \qquad \qquad \qquad + \cos^2 \alpha \sin \omega \sin (\varphi + \omega) \\ & + \cos^2 \alpha \cos \omega \cos (\varphi + \omega) - \cos^2 \alpha \cos \varphi \end{aligned} \right\} = 0.$$

Since

$$\cos \omega \sin (\varphi + \omega) - \sin \omega \cos (\varphi + \omega) = \sin \varphi$$

2

and

$$\sin \omega \sin (\varphi + \omega) + \cos \omega \cos (\varphi + \omega) = \cos \varphi,$$

this reduces to

$$\begin{aligned} & -\sin \alpha \cos \alpha \sin \varphi + \sin^2 \alpha \sin \varphi \tan \delta \\ & + \sin \omega \cos (\varphi + \omega) \tan \delta = 0; \end{aligned}$$

and since

$$\cos (\varphi + \omega) \sin \omega = \frac{1}{2} \sin (2\omega + \varphi) - \frac{1}{2} \sin \varphi,$$

this becomes

$$\tan \delta = \frac{2 \sin \alpha \cos \alpha \sin \varphi}{2 \sin^2 \alpha \sin \varphi + \sin (2\omega + \varphi) - \sin \varphi};$$

and since

$$\sin \alpha \cos \alpha = \frac{1}{2} \sin 2\alpha \quad \text{and} \quad 1 - 2 \sin^2 \alpha = \cos 2\alpha,$$

this reduces to

$$\tan \delta = \frac{\sin \varphi \sin 2\alpha}{\sin (2\omega + \varphi) - \sin \varphi \cos 2\alpha}. \quad (23)$$

This equation, therefore, expresses the condition that the “*sum of the moments of E, G, and R is zero.*”

Substituting  $\frac{\sin \delta}{\cos \delta}$  for  $\tan \delta$  in equation (23), clearing of fractions and factoring,

$$\sin \delta \sin (2\omega + \varphi) - \sin \delta \sin \varphi \cos 2\alpha = \sin \varphi \cos \delta \sin 2\alpha,$$

or

$$\sin \delta \sin (2\omega + \varphi) = \sin \varphi \cos \delta \sin 2\alpha + \sin \varphi \sin \delta \cos 2\alpha.$$

$$\text{Since } \cos \delta \sin 2\alpha + \sin \delta \cos 2\alpha = \sin (2\alpha + \delta),$$

this becomes

$$\sin \delta \sin (2\omega + \varphi) = \sin \varphi \sin (2\alpha + \delta). \quad (24)$$

In order to determine  $\omega$  it is only necessary to make  $\varepsilon = 0$  in equation (16*b*) express  $\sin (\varphi + \omega + \alpha + \delta)$  in terms of  $\sin$  and  $\cos (\varphi + \omega)$  and  $(\alpha + \delta)$ , and then the  $\sin$  and  $\cos$  of  $(\alpha + \delta)$  in terms of the  $\sin$  and  $\cos$  of  $\alpha$  and  $\delta$ . Making  $\varepsilon = 0$  in equation (16*b*), it becomes

$$\begin{aligned} \sin (\alpha + \omega) \cos (\alpha + \delta) \cos \omega \\ = \sin (\varphi + \omega + \alpha + \delta) [\cos (\varphi + \omega) \cos \alpha]. \end{aligned} \quad (24a)$$

Since

$$\begin{aligned} \sin (\varphi + \omega + \alpha + \delta) &= \sin (\varphi + \omega) \cos (\alpha + \delta) \\ &\quad + \cos (\varphi + \omega) \sin (\alpha + \delta) \\ \sin (\alpha + \delta) &= \sin \alpha \cos \delta + \cos \alpha \sin \delta \\ \cos (\alpha + \delta) &= \cos \alpha \cos \delta - \sin \alpha \sin \delta; \end{aligned}$$

hence

$$\begin{aligned} \sin (\varphi + \omega + \alpha + \delta) &= \sin (\varphi + \omega) \cos \alpha \cos \delta \\ &\quad - \sin (\varphi + \omega) \sin \alpha \sin \delta \\ &\quad + \cos (\varphi + \omega) \sin \alpha \cos \delta \\ &\quad + \cos (\varphi + \omega) \cos \alpha \sin \delta, \end{aligned}$$

and equation (24a) reduces to

$$\left. \begin{aligned} & \cos \omega \sin (\alpha + \omega) \cos \alpha \cos \delta \\ & \quad - \cos \omega \sin (\alpha + \omega) \sin \alpha \sin \delta \\ & - \cos^2 \alpha \cos (\varphi + \omega) \sin (\varphi + \omega) \cos \delta \\ & \quad + \cos \alpha \cos (\varphi + \omega) \sin (\varphi + \omega) \sin \alpha \sin \delta \\ & - \cos \alpha \cos^2 (\varphi + \omega) \sin \alpha \cos \delta \\ & - \cos^2 \alpha \cos^2 (\varphi + \omega) \sin \delta \end{aligned} \right\} = 0. \quad (24b)$$

Dividing by  $\cos \delta$ ,

$$\left. \begin{aligned} & \cos \alpha \cos \omega \sin (\alpha + \omega) \\ & \quad - \cos \omega \sin \alpha \sin (\alpha + \omega) \tan \delta \\ & - \cos^2 \alpha \cos (\varphi + \omega) \sin (\varphi + \omega) \\ & \quad + \cos \alpha \sin \alpha \cos (\varphi + \omega) \sin (\varphi + \omega) \tan \delta \\ & - \cos \alpha \sin \alpha \cos^2 (\varphi + \omega) \\ & - \cos^2 \alpha \cos^2 (\varphi + \omega) \tan \delta \end{aligned} \right\} = 0. \quad (24c)$$

Since

$\cos \alpha \cos \omega \sin (\alpha + \omega)$  equals, by expanding  $\sin (\alpha + \omega)$ ,  
 $\sin \alpha \cos \alpha \cos^2 \omega + \sin \omega \cos \omega \cos^2 \alpha$ , and likewise

$$\begin{aligned} - \cos \omega \sin \alpha \sin (\alpha + \omega) \tan \delta &= - \cos^2 \omega \sin^2 \alpha \tan \delta \\ - \cos \alpha \sin \alpha \cos \omega \sin \omega \tan \delta, \end{aligned}$$

equation (24c) becomes

$$\left. \begin{aligned} & - \sin \alpha \cos \alpha [\cos^2 (\varphi + \omega) - \cos^2 \omega] \\ & - \cos^2 \alpha [\sin (\varphi + \omega) \cos (\varphi + \omega) - \sin \omega \cos \omega] \\ & - [\cos^2 \alpha \cos^2 (\varphi + \omega) + \sin^2 \alpha \cos^2 \omega] \tan \delta \\ & + \sin \alpha \cos \alpha [\sin (\varphi + \omega) \cos (\varphi + \omega) \\ & \quad - \sin \omega \cos \omega] \tan \delta \end{aligned} \right\} = 0. \quad (24d)$$

Now

$$\cos^2(\varphi + \omega) - \cos^2 \omega = \frac{\cos 2(\varphi + \omega) - \cos 2\omega}{2},$$

which equals

$$\begin{aligned} & \frac{2 \sin \frac{1}{2} [2\omega - (2\varphi + 2\omega)] \sin \frac{1}{2} [2\omega + (2\varphi + 2\omega)]}{2} \\ &= \frac{2 \sin (-\varphi) \sin (2\omega + \varphi)}{2}, \end{aligned}$$

or  $-\sin (2\omega + \varphi) \sin \varphi,$

and

$$\begin{aligned} \sin (\varphi + \omega) \cos (\varphi + \omega) - \sin \omega \cos \omega \\ = \frac{1}{2} \sin 2(\varphi + \omega) - \frac{1}{2} \sin 2\omega; \end{aligned}$$

also,

$$\sin \alpha \cos \alpha = \frac{\sin 2\alpha}{2}, \text{ and } \cos^2 \alpha = \frac{\cos 2\alpha}{2} + \frac{1}{2}.$$

Hence, after multiplying by 2, equation (24d) reduces to

$$\left. \begin{aligned} & \sin 2\alpha \sin (2\omega + \varphi) \sin \varphi \\ & - \cos 2\alpha \frac{1}{2} \sin 2(\varphi + \omega) + \cos 2\alpha \frac{1}{2} \sin 2\omega \\ & - \frac{1}{2} \sin 2(\varphi + \omega) + \frac{1}{2} \sin 2\omega \\ & - \tan \delta \cos 2\alpha \cos^2 (\varphi + \omega) - \cos^2 (\varphi + \omega) \tan \delta \\ & - 2 \tan \delta \sin^2 \alpha \cos^2 \omega \\ & + \sin 2\alpha \sin (\varphi + \omega) \cos (\varphi + \omega) \tan \delta \\ & - \sin 2\alpha \sin \omega \cos \omega \tan \delta \end{aligned} \right\} = 0. \quad (24e)$$

Now

$$-2 \tan \delta \sin^2 \alpha \cos^2 \omega = [\text{since } \sin^2 \alpha = 1 - \cos^2 \alpha] \\ - [\cos^2 \omega - \cos^2 \alpha \cdot \cos^2 \omega] 2 \tan \delta,$$

which equals

$$- \underline{2 \cos^2 \omega \tan \delta} + 2 \tan \delta \cos^2 \alpha \cos^2 \omega.$$

Also,

$$- \frac{\cos 2\alpha \sin 2(\varphi + \omega)}{2} + \frac{\cos 2\alpha \sin 2\omega}{2} \\ = - \cos 2\alpha \left[ \frac{\sin 2(\varphi + \omega) - \sin 2\omega}{2} \right] \\ = - \frac{\cos 2\alpha [2 \sin \varphi \cos (2\omega + \varphi)]}{2} \\ = - \underline{\cos 2\alpha \cos (2\omega + \varphi) \sin \varphi},$$

and

$$- \frac{\sin 2(\varphi + \omega)}{2} + \frac{\sin 2\omega}{2} = - \frac{\sin 2(\varphi + \omega) - \sin 2\omega}{2} \\ = - \frac{2 \sin \frac{1}{2}(2\varphi + 2\omega - 2\omega) \cos \frac{1}{2}(2\varphi + 2\omega + 2\omega)}{2} \\ = - \underline{\sin \varphi \cos (2\omega + \varphi)},$$

and

$$- \tan \delta \cos 2\alpha \cos^2 (\varphi + \omega) + 2 \tan \delta \cos^2 \alpha \cos^2 \omega \\ = \left( \text{by making } \cos^2 \alpha = \frac{\cos 2\alpha}{2} + \frac{1}{2} \right) \\ - \tan \delta \cos 2\alpha [\cos^2 (\varphi + \omega) - \cos^2 \omega] + \tan \delta \cos^2 \omega, \\ \text{or } \underline{\tan \delta \cos 2\alpha \sin (2\omega + \varphi) \sin \varphi} + \tan \delta \cos^2 \omega,$$

$$\begin{aligned}
 \text{Also,} \quad & -\cos^2(\varphi + \omega) \tan \delta + \tan \delta \cos^2 \omega \\
 & = -\tan \delta [\cos^2(\varphi + \omega) - \cos^2 \omega] \\
 & = \underline{\sin \varphi \sin (2\omega + \varphi) \tan \delta}.
 \end{aligned}$$

Also,

$$\begin{aligned}
 & \tan \delta \sin 2\alpha \sin (\varphi + \omega) \cos (\varphi + \omega) \\
 & - \sin 2\alpha \sin \omega \cos \omega \tan \delta \\
 & = \tan \delta \sin 2\alpha [\sin (\varphi + \omega) \cos (\varphi + \omega) - \sin \omega \cos \omega] \\
 & = \tan \delta \sin 2\alpha \left[ \frac{\sin 2(\varphi + \omega) - \sin 2\omega}{2} \right] \\
 & = \underline{\tan \delta \sin 2\alpha \sin \varphi \cos (2\omega + \varphi)};
 \end{aligned}$$

and hence equation (24e) becomes

$$\left. \begin{aligned}
 & + \sin \varphi [\sin (2\omega + \varphi) \sin 2\alpha - \cos (2\omega + \varphi) \cos 2\alpha] \\
 & \quad - \sin \varphi \cos (2\omega + \varphi) \\
 & + \sin \varphi [\sin (2\omega + \varphi) \cos 2\alpha \\
 & \quad + \cos (2\omega + \varphi) \sin 2\alpha] \tan \delta \\
 & + \sin \varphi [\sin (2\omega + \varphi) \tan \delta] - 2 \cos^2 \omega \tan \delta
 \end{aligned} \right\} = 0, (24f)$$

and

$$\tan \delta = \frac{\sin \phi [\sin (2\omega + \phi) \sin 2\alpha - \cos (2\omega + \phi) \cos 2\alpha] - \sin \phi \cos (2\omega + \phi)}{2 \cos^2 \omega - \sin \phi [\sin (2\omega + \phi) \cos 2\alpha + \cos (2\omega + \phi) \sin 2\alpha] - \sin \phi \sin (2\omega + \phi)}.$$

By making  $\sin 2\alpha = 2 \sin \alpha \cos \alpha$  and  $\cos 2\alpha = 1 - 2 \sin^2 \alpha$  in the numerator, and  $\cos 2\alpha = 2 \cos \alpha \cos \alpha - 1$  and  $\sin 2\alpha = 2 \sin \alpha \cos \alpha$  in the denominator, this becomes

$$\begin{aligned}
 \tan \delta = & \frac{\sin \phi [\sin (2\omega + \phi) 2 \sin \alpha \cos \alpha - \cos (2\omega + \phi) + \cos (2\omega + \phi) 2 \sin^2 \alpha] - \sin \phi \cos (2\omega + \phi)}{2 \cos^2 \omega - \sin \phi [\sin (2\omega + \phi) 2 \cos^2 \alpha - \sin (2\omega + \phi) + \cos (2\omega + \phi) 2 \sin \alpha \cos \alpha] - \sin \phi \sin (2\omega + \phi)},
 \end{aligned}$$

or

$$\tan \delta = \frac{2 \sin \phi \sin \alpha [\sin (2\omega + \phi) \cos \alpha + \cos (2\omega + \phi) \sin \alpha] - 2 \sin \phi \cos (2\omega + \phi)}{2 \cos^2 \omega - 2 \sin \phi \cos \alpha [\sin (2\omega + \phi) \cos \alpha + \cos (2\omega + \phi) \sin \alpha]},$$

which reduces to

$$\tan \delta = \frac{\sin \varphi \sin \alpha \sin (2\omega + \varphi + \alpha) - \sin \varphi \cos (2\omega + \varphi)}{\cos^2 \omega - \sin \varphi \cos \alpha \sin (2\omega + \varphi + \alpha)}. \quad (24g)$$

Equating this value of  $\tan \delta$  with that in equation (23),

$$\begin{aligned} & \frac{\sin \varphi \sin \alpha \sin (2\omega + \varphi + \alpha) - \sin \varphi \cos (2\omega + \varphi)}{\cos^2 \omega - \sin \varphi \cos \alpha \sin (2\omega + \varphi + \alpha)} \\ &= \frac{\sin \varphi \sin 2\alpha}{\sin (2\omega + \varphi) - \sin \varphi \cos 2\alpha}. \end{aligned}$$

Dividing by  $\sin \varphi$ , clearing of fractions and dividing by  $\sin \alpha$ , also transposing, this becomes

$$\left. \begin{aligned} & \sin (2\omega + \varphi + \alpha) \sin (2\omega + \varphi) \\ & - \sin (2\omega + \varphi + \alpha) \sin \varphi \cos 2\alpha - \frac{\sin 2\alpha}{\sin \alpha} \cos^2 \omega \\ & + \frac{\sin 2\alpha}{\sin \alpha} \cos \alpha \sin (2\omega + \varphi + \alpha) \sin \varphi \\ & - \frac{\cos (2\omega + \varphi) [\sin (2\omega + \varphi) - \sin \varphi \cos 2\alpha]}{\sin \alpha} \end{aligned} \right\} = 0,$$

or

$$\left. \begin{aligned} & \sin (2\omega + \varphi + \alpha) \sin (2\omega + \varphi) \\ & - \sin \varphi \cos 2\alpha \sin (2\omega + \varphi + \alpha) - 2 \cos \alpha \cos^2 \omega \\ & + \sin \varphi 2 \cos^2 \alpha \sin (2\omega + \varphi + \alpha) \\ & - \frac{\cos (2\omega + \varphi) [\sin (2\omega + \varphi) - \sin \varphi \cos 2\alpha]}{\sin \alpha} \end{aligned} \right\} = 0.$$

Since  $2 \cos^2 \alpha - \cos 2\alpha = 1$ ,

this becomes

$$\sin (2\omega + \varphi + \alpha) [\sin (2\omega + \varphi) + \sin \varphi] - 2 \cos \alpha \cos^2 \omega - D = 0,$$

in which

$$D = \frac{\cos (2\omega + \varphi) [\sin (2\omega + \varphi) - \sin \varphi \cos 2\alpha]}{\sin \alpha},$$

or

$$\sin (2\omega + \varphi + \alpha) [2 \sin (\omega + \varphi) \cos \omega] - 2 \cos \alpha \cos^2 \omega - D = 0,$$

or

$$\sin (2\omega + \varphi + \alpha) \sin (\omega + \varphi) - \cos \alpha \cos \omega - \frac{D}{2 \cos \omega} = 0. \quad (25)$$

The formulæ for  $\omega$ ,  $\delta$ , and  $E$  can now be found in the simplest manner. Equation (25) is satisfied for  $2\omega + \varphi = 90^\circ$ . Hence,

$$\omega = 45^\circ - \frac{\varphi}{2}. \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Substituting this value in equation (23), it becomes

$$\begin{aligned} \tan \delta &= \frac{\sin \varphi \sin 2\alpha}{\sin (90 - \varphi + \varphi) - \sin \varphi \cos 2\alpha} \\ &= \frac{\sin \varphi \sin 2\alpha}{1 - \sin \varphi \cos 2\alpha}, \quad . \quad . \quad . \quad . \quad . \quad (27) \end{aligned}$$

or the equivalent, but more convenient expression for calculation,

$$\tan (\delta + \alpha) = \frac{\tan \alpha}{\tan^2 \left( 45^\circ - \frac{\varphi}{2} \right)}. \quad . \quad . \quad . \quad (28)$$

If, finally,  $\omega = 45^\circ - \frac{\varphi}{2}$  in equation (10), it becomes, remembering that  $k^2 = \frac{h^2}{\cos^2 \omega}$ ,

$$\begin{aligned} E &= \frac{\cos^2 \left( \varphi + 45^\circ - \frac{\varphi}{2} \right)}{\cos (\alpha + \delta)} \cdot \frac{h^2 \gamma}{2 \cos^2 \left( 45^\circ - \frac{\varphi}{2} \right)} \\ &= \frac{\cos^2 \left( 45^\circ + \frac{\varphi}{2} \right)}{\cos^2 \left( 45^\circ - \frac{\varphi}{2} \right)} \cdot \frac{h^2 \gamma}{2 \cos (\alpha + \delta)} \\ &= \frac{\sin^2 \left[ 90^\circ - \left( 45^\circ + \frac{\varphi}{2} \right) \right]}{\cos^2 \left( 45^\circ - \frac{\varphi}{2} \right)} \cdot \frac{h^2 \gamma}{2 \cos (\alpha + \delta)}; \end{aligned}$$

$$\text{hence } E = \tan^2 \left( 45^\circ - \frac{\varphi}{2} \right) \frac{h^2 \gamma}{2 \cos (\alpha + \delta)}, \quad \dots \quad (29)$$

or, from equation (28),

$$E = \frac{\tan \alpha}{\sin (\alpha + \delta)} \frac{h^2 \gamma}{2}. \quad \dots \quad (29a)$$

This last expression, however, when  $\alpha = 0$  takes the indeterminate form  $\frac{0}{0}$ .

The earth-pressure upon a portion of the wall reaching from the depth  $h_0$  to the depth  $H = h_0 + h$ , may be found

from equation (29) by substituting  $H^2 - h_0^2$  in place of  $h^2$ , as is evident from the following:

Suppose the wall to have a height  $H$ , then  $E_0 = C_0 \frac{H^2}{2} \gamma$ , and likewise for a height  $h$ ,

$$E_1 = C_0 \frac{h_0^2}{2} \gamma \therefore E = E_0 - E_1 = C_0 \frac{H^2 - h_0^2}{2} \gamma, \quad (29b)$$

$C_0$  representing the constant quantity.

From equation (29b)  $E = C(H^2 - h_0^2)$ ; hence  $dE = 2CHdH - 2Ch_0dh_0$ . Now let  $x$  equal the distance of the centre of pressure below the top of the wall, then

$$Ex = 2C \int_0^H H^2 dH - 2C \int_0^h h_0^2 dh,$$

$$\text{or} \quad C(H^2 - h_0^2)x = \frac{2}{3}CH^3 - \frac{2}{3}Ch_0^3,$$

$$\text{or} \quad x = \frac{2}{3} \frac{H^3 - h_0^3}{H^2 - h_0^2};$$

and if  $y$  = the distance from bottom,

$$y = \frac{1}{3} \frac{H^3 - 3Hh_0^2 + 2h_0^3}{H^2 - h_0^2}. \quad (30)$$

Equation (30) holds good when the earth-surface is loaded and the loading is equal to a distributed load of the height  $h_0$ . Still, even then,  $h_0$  is often so small that  $\frac{h}{3}$  can be substituted for it just as for unloaded earth-surface.

In all cases  $\delta$  is determined by equation (28).



but  $AG : AF :: AF : AD = h$ ,

$$\text{therefore } AG = \frac{\overline{AF}^2}{h} = h \tan^2 \left( 45^\circ - \frac{\varphi}{2} \right).$$

Now

$$\begin{aligned} HG = GD \sin \alpha &= (AG + AD) \sin \alpha \\ &= h \sin \alpha + h \tan^2 \left( 45^\circ - \frac{\varphi}{2} \right) \sin \alpha; \end{aligned}$$

$\tan AHG =$

$$\frac{h \tan^2 \left( 45^\circ - \frac{\varphi}{2} \right) \cos \alpha}{h \sin \alpha + h \tan^2 \left( 45^\circ - \frac{\varphi}{2} \right) \sin \alpha - h \tan^2 \left( 45^\circ - \frac{\varphi}{2} \right) \sin \alpha};$$

therefore

$$\tan AHG = \frac{\cos \alpha}{\sin \alpha} \tan^2 \left( 45^\circ - \frac{\varphi}{2} \right) = \cot \alpha \tan^2 \left( 45^\circ - \frac{\varphi}{2} \right).$$

From Fig. 4,  $\angle GDJ = \angle AHG$ ,  $\angle GDJ + \angle JGD = 90^\circ$ , and therefore

$$\tan JGD = \cot AHG = \tan \alpha \cot^2 \left( 45^\circ - \frac{\varphi}{2} \right) = \tan (\alpha + \delta),$$

or  $\angle JGD$  is the angle of the earth-pressure to the horizon.

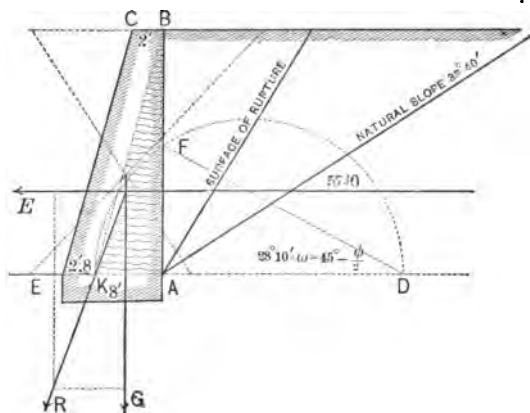
Since, now,  $\angle AIG = 90^\circ - \alpha - \delta$ ,

$$AH = \frac{\cos \alpha}{\cos (\alpha + \delta)} AG = h \tan^2 \left( 45^\circ - \frac{\varphi}{2} \right) \frac{\cos \alpha}{\cos (\alpha + \delta)},$$

and

$$\frac{1}{2} AH \cdot AB = \tan^2 \left( 45^\circ - \frac{\varphi}{2} \right) \frac{h^2}{2 \cos (\alpha + \delta)} = \frac{E}{\gamma}.$$

For a vertical wall the construction becomes much simpler. Draw, in Fig. 5,  $AD = h$  horizontally, then  $DF$  making the angle  $45^\circ - \frac{\varphi}{2}$  with  $AD$ . Draw through  $D$  and  $F$  a circle with centre in  $DA$  and continue it around to  $K$ .



**FIG. 5.**

then the  $\triangle ABK$  gives the intensity and distribution of the earth-pressure, while in direction it is horizontal.

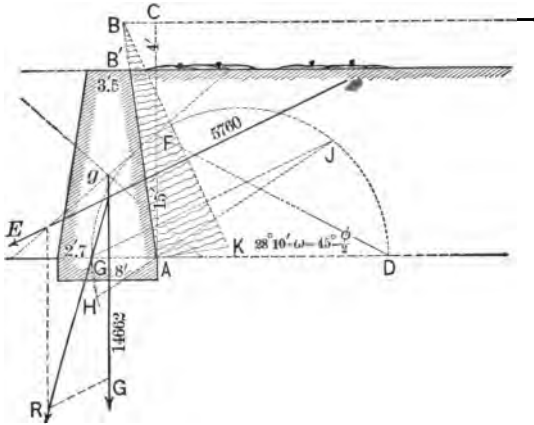
Hence  $E = \gamma \Delta ABK$ .

The proof is as follows (Fig. 5):

$$AK = \frac{AF}{AD} = \frac{h^2 \tan^2 \left( 45^\circ - \frac{\varphi}{2} \right)}{h} = h \tan^2 \left( 45^\circ - \frac{\varphi}{2} \right)$$

$$\frac{1}{2}AB \cdot AK = \frac{h^2}{2} \tan^2 \left( 45^\circ - \frac{\varphi}{2} \right) = \frac{E}{\gamma}.$$

As  $\alpha = 0$ , equation (28) gives  $\tan \delta = 0$ ;  $\therefore \delta = 0$  and  $E$  act normal to the surface of the wall.



**FIG. 6.**

Finally, in Fig. 6 is the construction for loaded earth-surface. The point of application of the earth-pressure is always found by drawing through the centre of gravity of  $\triangle ABK$  a parallel to  $AK$  and producing it to meet the wall. The proof for this construction is the same as that for Fig. 4.

## IV.

## EARTH SURFACE PARALLEL TO SURFACE OF REPOSE.

$$\varepsilon = \varphi.$$

For this case,

$$E = \frac{\cos^2 (\varphi - \alpha) l^2 \gamma}{\cos (\alpha + \delta) 2} = \left[ \frac{\cos (\varphi - \alpha)}{\cos \alpha} \right]^2 \frac{l^2 \gamma}{2 \cos (\alpha + \delta)}; \quad (20)$$

a formula which holds good for all values of  $\delta$ , and which for  $\delta = 0$  or  $\varphi$  gives results usually accepted in previous theories of retaining-walls. In order to find the proper values of  $\delta$  and  $\omega$ , equations (16*b*) and (22*b*) must be used.

In equation (22*b*) replace  $\sin (\varphi + \omega + \alpha + \delta)$  by  $\sin (\varphi + \omega + \alpha) \cos \delta + \cos (\varphi + \omega + \alpha) \sin \delta$ , and making  $\varepsilon = \varphi$  it becomes

$$\left. \begin{aligned} &+ \cos (\varphi + \omega) \cos (\varphi + \omega) \cos \delta \\ &\quad - \cos (\varphi + \omega) \sin (\varphi + \omega + \alpha) \cos \delta \sin \alpha \\ &- \cos (\varphi + \omega) \cos (\varphi + \omega + \alpha) \sin \delta \sin \alpha \end{aligned} \right\} =$$

$$= \left\{ \begin{aligned} &+ \cos (\alpha - \varphi) \cos (\alpha + \delta) \cos \varphi \\ &\quad - \cos (\alpha - \varphi) \sin (\varphi + \omega + \alpha) \sin \omega \cos \delta \\ &- \cos (\alpha - \varphi) \cos (\varphi + \omega + \alpha) \sin \delta \sin \omega; \end{aligned} \right.$$

dividing by  $\cos \delta$  and transposing,

$$\left. \begin{aligned} & - \frac{\cos(\alpha - \varphi) \cos(\alpha + \delta) \cos \varphi}{\cos \delta} \\ & + \cos(\alpha - \varphi) \sin(\varphi + \omega + \alpha) \sin \omega \\ & + \cos(\varphi + \omega) \cos(\varphi + \omega) \\ & - \cos(\varphi + \omega) \sin(\varphi + \omega + \alpha) \sin \alpha \end{aligned} \right\} =$$

$$= \left\{ \begin{aligned} & + \cos(\varphi + \omega) \cos(\varphi + \omega + \alpha) \frac{\sin \delta}{\cos \delta} \sin \alpha \\ & - \cos(\alpha - \varphi) \cos(\varphi + \omega + \alpha) \frac{\sin \delta}{\cos \delta} \sin \omega. \end{aligned} \right.$$

Since

$$\begin{aligned} & - \frac{\cos(\alpha - \phi) \cos(\alpha + \delta) \cos \phi}{\cos \delta} = - \frac{\cos(\alpha - \phi) \cos \phi (\cos \alpha \cos \delta - \sin \alpha \sin \delta)}{\cos \delta} \\ & = - \cos(\alpha - \phi) \cos \phi \cos \alpha + \cos(\alpha - \phi) \sin \alpha \frac{\sin \delta}{\cos \delta} \cos \phi, \end{aligned}$$

the above expression reduces to

$\tan \delta =$

$$\frac{\cos \alpha \cos(\alpha - \phi) \cos \phi - \cos \alpha \cos(\phi + \omega) \cos(\phi + \omega + \alpha) - \cos(\alpha - \phi) \sin \omega \sin(\phi + \omega + \alpha)}{\sin \alpha \cos(\alpha - \phi) \cos \phi - \sin \alpha \cos(\phi + \omega) \cos(\phi + \omega + \alpha) + \cos(\alpha - \phi) \sin \omega \cos(\phi + \omega + \alpha)}$$

and this equation fulfils the condition that the *sum of the moments of G, E, and R shall be zero.*

If equation (16b) is treated in a like manner, the resulting equation will fulfil the condition that the *sum of the forces parallel to the surface of rupture shall equal zero.* Making  $\varepsilon = \varphi$  in equation (16b), it reduces to

$$\begin{aligned} & \sin(\alpha + \omega) \cos(\varphi + \omega) \cos(\alpha + \delta) \\ & - \sin(\varphi + \alpha + \omega + \delta) \cos(\varphi + \omega) \cos(\alpha - \varphi) = 0, \\ & \quad \quad \quad \S \end{aligned}$$

or

$$\sin(\alpha + \omega) \cos(\alpha + \delta) - \sin(\varphi + \omega + \alpha) \cos(\alpha - \varphi) \cos \delta \\ - \cos(\varphi + \omega + \alpha) \cos(\alpha - \varphi) \sin \delta = 0,$$

or

$$\frac{\sin(\alpha + \omega) \cos \alpha \cos \delta}{\cos \delta} - \frac{\sin(\alpha + \omega) \sin \alpha \sin \delta}{\cos \delta} - \\ \sin(\varphi + \omega + \alpha) \cos(\alpha - \varphi) - \frac{\cos(\varphi + \omega + \alpha) \cos(\alpha - \varphi) \sin \delta}{\cos \delta} = 0;$$

therefore

$$\tan \delta = \frac{\cos \alpha \sin(\alpha + \omega) - \sin(\varphi + \omega + \alpha) \cos(\alpha - \varphi)}{\sin(\alpha + \omega) \sin \alpha + \cos(\varphi + \omega + \alpha) \cos(\alpha - \varphi)}.$$

Setting both values of  $\tan \delta$  equal to each other and clearing of fractions, the following expression is obtained:

$$\begin{aligned} & + \cos \alpha \cos \varphi \sin \alpha \sin(\omega + \alpha) \cos(\alpha - \varphi) \\ & - \cos \alpha \sin \alpha \sin(\omega + \alpha) \cos(\omega + \varphi) \cos(\omega + \varphi + \alpha) \\ & - \sin \omega \sin \alpha \sin(\omega + \alpha) \cos(\alpha - \varphi) \sin(\varphi + \omega + \alpha) \\ & + \cos \alpha \cos \varphi \cos(\alpha - \varphi) \cos(\varphi + \omega + \alpha) \cos(\alpha - \varphi) \\ & - \cos \alpha \cos(\varphi + \omega) \cos^2(\varphi + \omega + \alpha) \cos(\alpha - \varphi) \\ & - \sin \omega \cos^2(\alpha - \varphi) \sin(\varphi + \omega + \alpha) \cos(\varphi + \omega + \alpha) \end{aligned}$$

for the first member of the equation, and

$$\begin{aligned} & + \cos \alpha \cos \varphi \sin \alpha \sin(\omega + \alpha) \cos(\alpha + \varphi) \\ & - \sin \alpha \cos \alpha \sin(\omega + \alpha) \cos(\omega + \varphi) \cos(\varphi + \omega + \alpha) \\ & + \sin \omega \cos \alpha \sin(\omega + \alpha) \cos(\alpha - \varphi) \cos(\varphi + \omega + \alpha) \\ & - \sin \alpha \cos \varphi \cos^2(\alpha - \varphi) \sin(\varphi + \omega + \alpha) \\ & + \sin \alpha \cos(\varphi + \omega) \cos(\varphi + \omega + \alpha) \cos(\alpha - \varphi) \sin(\varphi + \omega + \alpha) \\ & - \sin \omega \cos^2(\alpha - \varphi) \cos(\varphi + \omega + \alpha) \sin(\varphi + \omega + \alpha) \end{aligned}$$

for the second member,

The first terms, second terms, and sixth terms cancel. Divide the equation by  $\cos (\alpha - \varphi)$ . Terms number 3 combined give

$$- \sin \omega \sin (\omega + \alpha) [\sin \alpha \sin (\phi + \omega + \alpha) + \cos \alpha \cos (\phi + \omega + \alpha)],$$

which becomes

$$- \sin \omega \sin (\omega + \alpha) \cos (\varphi + \omega).$$

Terms number 5 combined give

$$- \cos (\phi + \omega) \cos (\phi + \omega + \alpha) [\cos \alpha \cos (\phi + \omega + \alpha) + \sin \alpha \sin (\phi + \omega + \alpha)],$$

which becomes

$$- \cos (\varphi + \omega + \alpha) \cos (\varphi + \omega) \cos (\varphi + \omega).$$

Terms number 4 combined give

$$+ \cos \varphi \cos (\alpha - \varphi) [\cos \alpha \cos (\varphi + \omega + \alpha) + \sin \alpha \sin (\varphi + \omega + \alpha)],$$

which becomes

$$+ \cos \varphi \cos (\alpha - \varphi) \cos (\varphi + \omega),$$

and hence, after dividing by  $\cos (\varphi + \omega)$ , the equation above reduces to

$$\cos (\alpha - \varphi) \cos \varphi - \cos (\varphi + \omega + \alpha) \cos (\varphi + \omega) - \sin (\omega + \alpha) \sin \omega = 0, \quad (31)$$

and this equation is fulfilled for

$$\omega = 90^\circ - \varphi. \quad . \quad . \quad . \quad . \quad (32)$$

In order to find that value of  $\delta$  which satisfies all conditions of equilibrium, substitute the above value of  $\omega$  in the first expression for  $\tan \delta$  and obtain  $\frac{0}{0}$ . If, according to

the method for discussing indeterminate fractions, the first differentials of the numerator and denominator and their ratio are found, and  $\omega$  made equal to  $90^\circ - \varphi$ , the value of  $\tan \delta$  will be found.

The differential of the numerator is

$$d[-\cos \alpha \cos (\varphi + \omega) \cos (\varphi + \omega + \alpha) - \cos (\alpha - \varphi) \sin \omega \sin (\varphi + \omega + \alpha)],$$

which equals

$$\left\{ \begin{array}{l} + \cos \alpha \cos (\varphi + \omega + \alpha) \sin (\varphi + \omega) \\ + \cos \alpha \cos (\varphi + \omega) \sin (\varphi + \omega + \alpha) \\ - \cos (\alpha - \varphi) \sin (\varphi + \omega + \alpha) \cos \omega \\ - \cos (\alpha - \varphi) \sin \omega \cos (\varphi + \omega + \alpha) \end{array} \right\} d\omega.$$

Substituting for  $\omega$ ,  $90^\circ - \varphi$ , this becomes

$$\left\{ \begin{array}{l} + \cos \alpha \cos (\varphi + 90^\circ - \varphi + \alpha) \sin (\varphi + 90^\circ - \varphi) \\ + \cos \alpha \cos (\varphi + 90^\circ - \varphi) \sin (\varphi + 90^\circ - \varphi + \alpha) \\ - \cos (\alpha - \varphi) \sin (\varphi + 90^\circ - \varphi + \alpha) \cos (90^\circ - \varphi) \\ + \cos (\alpha - \varphi) \sin (90^\circ - \varphi) \cos (\varphi + 90^\circ - \varphi + \alpha) \end{array} \right\} d\omega.$$

As the second term reduces to zero, this becomes

$$[\cos \alpha \sin \alpha - \cos (\alpha - \varphi) \cos \alpha \sin \varphi + \cos (\alpha - \varphi) \cos \varphi \sin \alpha] d\omega,$$

or

$$\left[ \frac{\sin 2\alpha}{2} - \cos (\alpha - \varphi) (\cos \alpha \sin \varphi - \cos \varphi \sin \alpha) \right] d\omega,$$

or

$$\begin{aligned} & \left[ \frac{\sin 2\alpha}{2} - \cos (\alpha - \varphi) \sin (\varphi - \alpha) \right] d\omega \\ &= \left[ \frac{\sin 2\alpha}{2} + \frac{\sin 2(\varphi - \alpha)}{2} \right] d\omega, \end{aligned}$$

or

$$\left[ \frac{2 \sin \frac{1}{2}(2\varphi - 2\alpha + 2\alpha) \cos \frac{1}{2}(2\varphi - 2\alpha - 2\alpha)}{2} \right] d\omega,$$

which equals  $\sin \varphi \cos(\varphi - 2\alpha) d\omega$ .

The differential of the denominator is

$$\left\{ \begin{array}{l} + \sin \alpha \cos(\varphi + \omega + \alpha) \sin(\varphi + \omega) \\ + \sin \alpha \cos(\varphi + \omega) \sin(\varphi + \omega + \alpha) \\ + \cos(\alpha - \varphi) \cos(\varphi + \omega + \alpha) \cos \omega \\ + \cos(\alpha - \varphi) \sin \omega \sin(\varphi + \omega + \alpha) \end{array} \right\} d\omega.$$

Substituting  $90^\circ - \varphi$  for  $\omega$ , and this becomes

$$[\sin \alpha \sin \alpha + \cos(\alpha - \varphi) \sin \alpha \sin \varphi + \cos(\alpha - \varphi) \cos \varphi \cos \alpha] d\omega,$$

or

$$[\sin^2 \alpha + \cos(\alpha - \varphi) (\sin \varphi \sin \alpha + \cos \varphi \cos \alpha)] d\omega,$$

or

$$\begin{aligned} & [1 - \cos^2 \alpha + \cos(\alpha - \varphi) \cos(\alpha - \varphi)] d\omega \\ &= \left[ 1 - \frac{\cos 2\alpha}{2} - \frac{1}{2} + \frac{\cos 2(\alpha - \varphi)}{2} + \frac{1}{2} \right] d\omega, \end{aligned}$$

or

$$[1 - \sin \varphi \sin(\varphi - 2\alpha)] d\omega;$$

therefore

$$\tan \delta = \frac{\sin \varphi \cos(\varphi - 2\alpha)}{1 - \sin \varphi \sin(\varphi - 2\alpha)}. \quad \cdot \cdot \quad (33)$$

To find an expression for the  $\sin \delta$ , clear equation (33)

of fractions and deduce  $\tan \delta - \tan \delta \sin \varphi \sin (\varphi - 2\alpha)$   
 $= \sin \varphi \cos (\varphi - 2\alpha)$ . Multiplying by  $\cos \delta$ ,

$$\sin \delta - \sin \delta \sin \varphi \sin (\varphi - 2\alpha) = \sin \varphi \cos (\varphi - 2\alpha) \cos \delta,$$

or

$$\sin \delta = \sin \varphi [\sin \delta \sin (\varphi - 2\alpha) + \cos (\varphi - 2\alpha) \cos \delta];$$

therefore

$$\sin \delta = \sin \varphi \cos (2\alpha - \varphi + \delta), \quad . \quad . \quad (34)$$

from which the results of III. can be deduced.

If the earth-surface is parallel to the surface of repose, or makes the angle  $\varphi$  with the horizontal, then, under the assumption of a plane surface of rupture,  $\delta = \varphi$  only when the wall is vertical (make  $\alpha = 0$  in equation (33), then  $\tan \delta = \tan \varphi$ ;  $\therefore \delta = \varphi$ ), and  $\delta = 0$  only when the angle of the wall with the vertical  $\alpha = 45^\circ + \frac{\varphi}{2}$ .

As it is often more convenient in determining the direction of the earth-pressure to know the angle  $(\alpha + \delta)$  of  $E$  with the horizon,  $\tan (\alpha + \delta)$  may be expressed in terms of  $\tan \alpha$  and  $\tan \delta$ , remembering that

$$\cos \alpha - \sin \varphi \sin (\varphi - \alpha) = \cos \varphi \cos (\varphi - \alpha),$$

and hence

$$\tan (\alpha + \delta) = \frac{\sin \alpha + \sin \varphi \cos (\varphi - \alpha)}{\cos \varphi \cos (\varphi - \alpha)}. \quad . \quad (34a)$$

With reference to a limited portion of wall which does

not reach as far as the surface, and with reference to loaded earth-surface, the same remarks hold good as in III.

Instead of formulæ (20) and (33) or (34), the following construction may be used:

Draw through  $A$ , Fig. 7, a parallel to the earth-surface,

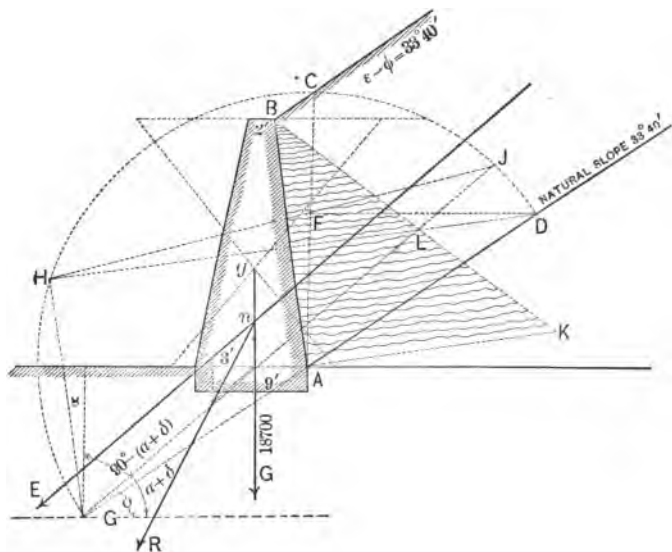


FIG. 7.

and with  $AC$  as a radius describe the circle  $ADG$ . Draw  $DF$  horizontal and  $GH$  parallel to  $AB$ , and then the straight line  $HfJ$ . Then the direction of the earth-pressure is  $GJ$ ; and if  $AK$  is made perpendicular to  $AB$  and equal to  $HF$ ,  $E = \gamma \Delta ABK$ , and the triangle gives the distribution of the pressure. The point of application is found by drawing through the centre of gravity of the triangle a perpendicular to  $AB$ .

The proof of this construction is as follows :

Conceive  $HD$  drawn, and its intersection with  $GJ$  to be at  $L$ . Then from the notation of Fig. 3, where  $\epsilon = \varphi$ ,

$$FD = AD \cos \varphi, \quad HD = 2AD \cos (\varphi - \alpha).$$

Since, now,  $\angle JLD = \angle JHD + \varphi - \alpha$ , by expressing  $\tan JLD$  by  $\tan$  of  $JHD$  and  $\varphi - \alpha$ , after reducing,

$$\tan JLD = \frac{\cos \varphi \sin (2\alpha - \varphi) + \sin 2(\varphi - \alpha)}{1 + \cos 2(\varphi - \alpha) - \cos \varphi \cos (2\alpha - \varphi)},$$

or

$$\tan JLD = \frac{\sin \varphi \cos (\varphi - 2\alpha)}{1 + \sin \varphi \sin (\varphi - 2\alpha)} = \tan \delta.$$

Since  $HD$  is perpendicular to  $AB$ , the earth-pressure has the direction  $GJ$ . Further,

$$HF = \frac{FD \sin \alpha}{\sin (\alpha + \delta - \varphi)} = \frac{\sin \alpha \cos \varphi}{\sin (\alpha + \delta - \varphi)} AD,$$

$$AD = \frac{l \cos (\varphi - \alpha)}{\cos \varphi}, \text{ or, with reference to the value of } FD.$$

$$\triangle ABK = \frac{\cos (\varphi - \alpha) \sin \alpha l^2}{\sin (\alpha + \delta - \varphi) 2}, \text{ and since from equation (34) } \sin (\alpha + \delta - \varphi) \cos (\varphi - \alpha) = \sin \alpha \cos (\alpha + \delta),$$

$$\triangle ABK = \frac{\cos^2 (\varphi - \alpha) l^2}{\cos (\alpha + \delta) 2} = \frac{E}{\gamma}.$$

RECAPITULATION OF FORMULÆ.

Inclined earth-surface, plane :

$$n = \sqrt{\frac{\sin (\varphi + \delta) \sin (\varphi - \varepsilon)}{\cos (\alpha + \delta) \cos (\alpha - \varepsilon)}}. \quad . \quad . \quad (18)$$

The  $\tan \delta$  deduced from formulæ (22*b*) and (16*l*):

$$\tan \delta = \frac{\sin (2\alpha - \varepsilon) - K \sin 2(\alpha - \varepsilon)}{K - \cos (2\alpha - \varepsilon) + K \cos 2(\alpha - \varepsilon)},$$

in which

$$K = \frac{\cos \varepsilon - \sqrt{\cos^2 \varepsilon - \cos^2 \varphi}}{\cos^2 \varphi},$$

$$E = \left[ \frac{\cos (\varphi - \alpha)}{(n + 1) \cos \alpha} \right]^2 \frac{h^2 \gamma}{2 \cos (\alpha + \delta)}. \quad . \quad . \quad (19)$$

Earth-surface parallel to natural slope :

$$\varepsilon = \varphi ;$$

$$E = \left[ \frac{\cos (\varphi - \alpha)}{\cos \alpha} \right]^2 \frac{h^2 \gamma}{2 \cos (\alpha + \delta)}; \quad . \quad . \quad (20)$$

$$\omega = 90^\circ - \varphi; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$$\tan (\alpha + \delta) = \frac{\sin \alpha + \sin \varphi \cos (\varphi - \alpha)}{\cos \varphi \cos (\varphi - \alpha)}; \quad . \quad . \quad (34a)$$

$$\tan \delta = \frac{\sin \varphi \cos (\varphi - 2\alpha)}{1 - \sin \varphi \sin (\varphi - 2\alpha)}. \quad . \quad . \quad . \quad (33)$$



NOMENCLATURE.

Height of wall .....	$H$
Thickness at base.....	$b$
Thickness at top.....	$b'$
Batter in inches per foot of $H$ on front face...	$d$
Weight per cubic foot.....	$W$
Total weight of wall .....	$G$
Angle of repose of earth.....	$\varphi$
Angle made by surface of rupture with vertical	$\omega$
Weight of cubic foot of earth.....	$\gamma$
Total thrust of earth against wall.....	$E$
Angle made with the horizontal by the surface of the earth.....	$\varepsilon$
Angle made by rear face of wall with the ver- tical.....	$\alpha$
Angle made with normal by $E$ .....	$\delta$
Dist. of point where the resultant pressure cuts the base from the front edge of the wall..	$q$
The resultant pressure due to $E$ and $G$ .....	$R$

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NOTE.

FOR the translation of Prof. Weyrauch's paper the writer is indebted to the labor of Prof. A. J. Du Bois, of the Sheffield Scientific School, Yale College, who had copies printed by the electric-pen process. However, only the leading equations of Prof. Weyrauch were given; hence a great deal of labor has been devoted to expanding, verifying, and filling in the intermediate steps of the work, and this nucleus of the mathematical part alone has grown to about double the original quantity.

M. A. H.



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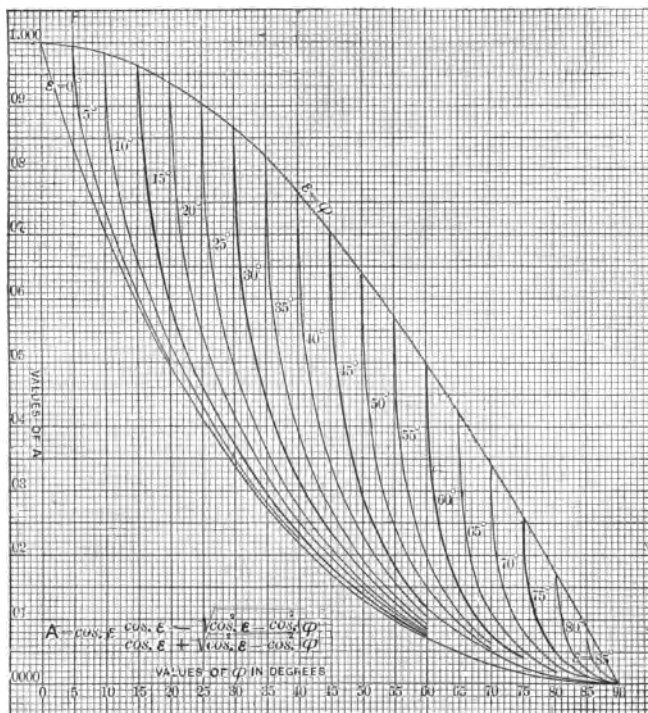
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DIAGRAM I.





## TABLES.

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*Table I* contains the crushing-strengths and the average weights of stone likely to be used in the construction of retaining-walls and foundations; also the average weights of different earths.

*Table II* contains the coefficients of friction, limiting angles of friction, and the reciprocals of the coefficients of friction for various substances.

*Tables III, IV, and V* contain the values of the coefficients [see equation (1')] (*B*), (*C*), (*D*) and (*E*), where

$$(B) = \frac{\cos (\epsilon - \alpha)}{\cos ^2 \alpha \cos \epsilon}, \quad (C) = \sin ^2 \alpha, \quad (D) = \left\{ \frac{\cos (\epsilon - \alpha)}{\cos \epsilon} \right\}^2$$

and 
$$(E) = 2 \sin \alpha \sin \epsilon \frac{\cos (\epsilon - \alpha)}{\cos \epsilon}.$$

The tables were computed with a Thacher calculating instrument and checked by means of diagrams. It is believed that they are correct to the second place of decimals; an error in the third place of decimals does not affect the results for practical purposes.

*Table VI* contains the natural sines, cosines and tangents.

TABLE I.  
VALUES OF *W*.

Name of Substance.	Crushing Lds. in tons per sq. ft.	Average weight in lbs. per cu. ft.
Alabaster.....	.....	144
Brick, best pressed.....	40 to 300	150
“ common hard.....	.....	125
“ soft inferior.....	.....	100
Chalk.....	20 to 30	156
Cement, loose.....	.....	49.6 to 102
Flint.....	.....	162
Feldspar.....	.....	166
Granite.....	300 to 1200	170
Gneiss.....	.....	168
Greenstone, trap.....	.....	187
Hornblende, black.....	.....	203
Limestones and Marbles, ordinary.....	250 to 1000	{ 164.4 168
Mortar, hardened.....	.....	103
Quartz, common.....	.....	165
Sandstone.....	150 to 550	151
Shales.....	.....	162
Slate.....	400 to 800	175
Soapstone.....	.....	170

VALUES OF *γ*.

Name of Substance.	Average weight in lbs. per cu. ft.
Earth, common loam, loose.....	72 to 80
“ “ “ shaken.....	82 “ 93
“ “ “ rammed moderately.....	90 “ 100
Gravel.....	90 “ 106
Sand.....	90 “ 106
Soft flowing mud.....	104 “ 120
Sand perfectly wet.....	118 “ 129

TABLE II.

\* ANGLES AND COEFFICIENTS OF FRICTION.

	$\tan \phi$ .	$\phi$	$\frac{1}{\tan \phi}$
Dry masonry and brickwork	0.6 to 0.7	31° to 35°	1.67 to 1.43
Masonry and brickwork with damp mortar.....	0.74	36½°	1.35
Timber on stone.....	about 0.4	22°	2.5
Iron on stone ... ..	0.7 to 0.3	35° to 16½°	1.43 to 3.33
Timber on timber.....	0.5 " 0.2	26½° " 11½°	2 " 5
Timber on metals.....	0.6 " 0.2	31° " 11½°	1.67 " 5
Metals on metals.....	0.25 " 0.15	14° " 8½°	4 " 6.67
Masonry on dry clay.....	0.51	27°	1.96
" " moist clay.....	0.33	18½°	3.
Earth on earth.....	0.25 to 1.0	14° to 45°	4 to 1
Earth on earth, dry sand, clay, and mixed earth....	0.38 " 0.75	21° " 37°	2.63 " 1.33
Earth on earth, damp clay .	1.0	45°	1
Earth on earth, wet clay. .	0.31	17°	3.23
Earth on earth, shingle and gravel.....	0.81	39° to 48°	1.23 to 0.9

\* From Rankine's Applied Mechanics.

TABLE III.

$\epsilon$	$\alpha = 5^\circ$	$\alpha = 6^\circ$	$\alpha = 7^\circ$	$\alpha = 8^\circ$	$\alpha = 9^\circ$
	(B)	(B)	(B)	(B)	(B)
0	1.004	1.005	1.007	1.010	1.012
5	1.012	1.015	1.018	1.022	1.026
10	1.019	1.024	1.029	1.035	1.040
15	1.027	1.034	1.041	1.048	1.055
20	1.036	1.044	1.052	1.062	1.071
25	1.045	1.055	1.065	1.076	1.088
30	1.055	1.066	1.079	1.092	1.105
35	1.065	1.079	1.094	1.109	1.124
40	1.078	1.094	1.111	1.129	1.147
45	1.093	1.111	1.131	1.152	1.173
	(C)	(C)	(C)	(C)	(C)
	0.008	0.011	0.015	0.019	0.024

TABLE IV.

$\epsilon$	$\alpha = 5^\circ$	$\alpha = 6^\circ$	$\alpha = 7^\circ$	$\alpha = 8^\circ$	$\alpha = 9^\circ$
	(D)	(D)	(D)	(D)	(D)
0	0.992	0.989	0.985	0.981	0.976
5	1.008	1.008	1.006	1.005	1.003
10	1.023	1.026	1.028	1.030	1.031
15	1.040	1.046	1.051	1.056	1.060
20	1.057	1.066	1.075	1.084	1.092
25	1.075	1.089	1.102	1.114	1.125
30	1.096	1.113	1.130	1.147	1.163
35	1.118	1.140	1.164	1.183	1.204
40	1.144	1.172	1.199	1.226	1.253
45	1.174	1.208	1.242	1.276	1.309

TABLE V.

$\epsilon$	$\alpha = 5^\circ$	$\alpha = 6^\circ$	$\alpha = 7^\circ$	$\alpha = 8^\circ$	$\alpha = 9^\circ$
	(E)	(E)	(E)	(E)	(E)
0	0	0	0	0	0
5	0.015	0.018	0.021	0.024	0.027
10	0.031	0.037	0.043	0.049	0.055
15	0.046	0.055	0.065	0.074	0.083
20	0.061	0.074	0.086	0.099	0.112
25	0.076	0.092	0.108	0.124	0.140
30	0.091	0.110	0.130	0.149	0.169
35	0.106	0.128	0.151	0.174	0.197
40	0.120	0.145	0.172	0.198	0.225
45	0.134	0.162	0.192	0.222	0.253

TABLE III—Continued.

$\epsilon$	$\alpha = 10^\circ$	$\alpha = 11^\circ$	$\alpha = 12^\circ$	$\alpha = 13^\circ$	$\alpha = 14^\circ$
	(B)	(B)	(B)	(B)	(B)
0	1.015	1.019	1.022	1.026	1.031
5	1.031	1.037	1.041	1.047	1.053
10	1.046	1.055	1.061	1.068	1.076
15	1.063	1.073	1.081	1.090	1.100
20	1.081	1.092	1.103	1.112	1.125
25	1.099	1.112	1.124	1.136	1.150
30	1.119	1.135	1.151	1.163	1.179
35	1.141	1.159	1.175	1.195	1.211
40	1.166	1.186	1.205	1.225	1.245
45	1.195	1.218	1.240	1.263	1.288
	(C)	(C)	(C)	(C)	(C)
	0.030	0.036	0.043	0.051	0.029

TABLE IV—Continued.

$\epsilon$	$\alpha = 10^\circ$	$\alpha = 11^\circ$	$\alpha = 12^\circ$	$\alpha = 13^\circ$	$\alpha = 14^\circ$
	(D)	(D)	(D)	(D)	(D)
0	0.970	0.964	0.957	0.950	0.942
5	1.000	0.997	0.993	0.988	0.983
10	1.031	1.031	1.030	1.028	1.026
15	1.064	1.067	1.069	1.061	1.072
20	1.099	1.105	1.110	1.116	1.121
25	1.136	1.147	1.156	1.165	1.173
30	1.178	1.194	1.204	1.220	1.232
35	1.224	1.244	1.262	1.281	1.300
40	1.291	1.304	1.328	1.353	1.377
45	1.342	1.375	1.407	1.438	1.469

TABLE V—Continued.

$\epsilon$	$\alpha = 10^\circ$	$\alpha = 11^\circ$	$\alpha = 12^\circ$	$\alpha = 13^\circ$	$\alpha = 14^\circ$
	(E)	(E)	(E)	(E)	(E)
0	0	0	0	0	0
5	0.030	0.032	0.036	0.039	0.042
10	0.061	0.067	0.073	0.079	0.085
15	0.093	0.102	0.111	0.119	0.130
20	0.124	0.137	0.150	0.163	0.175
25	0.156	0.173	0.189	0.205	0.221
30	0.188	0.208	0.216	0.248	0.269
35	0.220	0.244	0.268	0.292	0.316
40	0.252	0.280	0.308	0.336	0.365
45	0.284	0.316	0.349	0.382	0.415

TABLE III—Continued.

$\epsilon$	$\alpha = 15^\circ$	$\alpha = 16^\circ$	$\alpha = 17^\circ$	$\alpha = 18^\circ$	$\alpha = 20^\circ$
	(B)	(B)	(B)	(B)	(B)
0	1.035	1.040	1.048	1.051	1.062
5	1.059	1.066	1.076	1.081	1.098
10	1.084	1.093	1.104	1.112	1.132
15	1.110	1.120	1.134	1.138	1.168
20	1.135	1.149	1.165	1.177	1.218
25	1.165	1.179	1.197	1.211	1.245
30	1.195	1.212	1.233	1.248	1.288
35	1.229	1.249	1.272	1.291	1.339
40	1.268	1.291	1.317	1.340	1.389
45	1.313	1.338	1.369	1.393	1.451
	(C)	(C)	(C)	(C)	(C)
	0.067	0.076	0.086	0.095	0.117

TABLE IV—Continued.

$\epsilon$	$\alpha = 15^\circ$	$\alpha = 16^\circ$	$\alpha = 17^\circ$	$\alpha = 18^\circ$	$\alpha = 20^\circ$
	(D)	(D)	(D)	(D)	(D)
0	0.933	0.924	0.915	0.905	0.833
5	0.977	0.971	0.964	0.957	0.940
10	1.023	1.018	1.016	1.011	1.000
15	1.073	1.073	1.071	1.069	1.068
20	1.124	1.127	1.129	1.131	1.132
25	1.181	1.188	1.194	1.200	1.208
30	1.244	1.256	1.266	1.276	1.293
35	1.316	1.332	1.348	1.363	1.390
40	1.400	1.422	1.444	1.465	1.505
45	1.500	1.530	1.559	1.588	1.648

TABLE V—Continued.

$\epsilon$	$\alpha = 15^\circ$	$\alpha = 16^\circ$	$\alpha = 17^\circ$	$\alpha = 18^\circ$	$\alpha = 20^\circ$
	(E)	(E)	(E)	(E)	(E)
0	0	0	0	0	0
5	0.045	0.047	0.050	0.053	0.058
10	0.091	0.097	0.102	0.108	0.119
15	0.139	0.148	0.157	0.165	0.183
20	0.188	0.200	0.213	0.225	0.249
25	0.238	0.254	0.270	0.277	0.318
30	0.289	0.309	0.329	0.349	0.389
35	0.341	0.365	0.390	0.414	0.463
40	0.394	0.423	0.452	0.481	0.539
45	0.448	0.482	0.516	0.551	0.620

## TABLE VI.

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NATURAL SINES, COSINES, TANGENTS  
AND COTANGENTS.

	0°		1°		2°		3°		4°		
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00173	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00843	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
	89°	88°	87°	86°	85°						

	5°		6°		7°		8°		9°		
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99403	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98949	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14839	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14983	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10193	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10222	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
	84°		83°		82°		81°		80°		

	10°		11°		12°		13°		14°		
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96830	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96823	31
30	.18224	.98325	.19937	.97993	.21644	.97630	.23345	.97237	.25038	.96816	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96809	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96802	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96795	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96788	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96781	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96774	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96767	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96760	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96753	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96746	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96739	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96732	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96725	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96718	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96711	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96704	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96697	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96690	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96683	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25602	.96676	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96669	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96662	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96655	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96648	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96641	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96634	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96627	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96620	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96613	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96606	0
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
	79°		78°		77°		76°		75°		

	15°		16°		17°		18°		19°		
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.25882	.96693	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95623	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95615	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95606	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94371	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94833	.33381	.94264	30
31	.26752	.96355	.28430	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30293	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28653	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29210	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
	74°		73°		72°		71°		70°		

	20°		21°		22°		23°		24°		
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93960	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40089	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90803	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93473	.37164	.92838	.38779	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38913	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
	69°	68°	67°	66°	65°						

	25°		26°		27°		28°		29°		
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.42362	.90631	.43837	.89679	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42368	.90618	.43863	.89667	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42375	.90606	.43889	.89654	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42381	.90594	.43916	.89641	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42387	.90582	.43942	.89628	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89616	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42400	.90557	.43994	.89603	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42406	.90545	.44020	.89590	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42413	.90532	.44046	.89577	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42419	.90520	.44072	.89564	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42425	.90507	.44098	.89552	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42532	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87122	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43732	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
	64°		63°		62°		61°		60°		

	30°		31°		32°		33°		34°		
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82823	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83644	.56256	.82676	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82660	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82644	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82628	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82612	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82596	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82579	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82563	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82547	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82531	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82515	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82499	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82483	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82467	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82451	32
29	.50729	.86177	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82435	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82419	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82403	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83357	.56689	.82387	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83341	.56713	.82371	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82355	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82339	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82323	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82307	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82291	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55413	.83244	.56856	.82275	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55437	.83228	.56880	.82259	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55461	.83212	.56904	.82243	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55485	.83196	.56928	.82227	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82211	17
44	.51104	.85955	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82195	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82179	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82163	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82147	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55629	.83099	.57071	.82131	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55653	.83083	.57095	.82115	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55677	.83067	.57119	.82099	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55701	.83051	.57143	.82083	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55725	.83035	.57167	.82067	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83019	.57191	.82051	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55774	.83003	.57215	.82035	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82987	.57239	.82019	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82971	.57263	.82003	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82955	.57287	.81987	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82939	.57311	.81971	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82923	.57335	.81955	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82907	.57359	.81939	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
	59°		58°		57°		56°		55°		

	35°		36°		37°		38°		39°		
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77642	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61910	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77825	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
	54°		53°		52°		51°		50°		

	40°		41°		42°		43°		44°	
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711
12	.64546	.76379	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610
17	.64657	.76285	.65978	.75145	.67280	.73983	.68561	.72797	.69821	.71590
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69863	.71549
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488
23	.64790	.76173	.66110	.75030	.67409	.73865	.68688	.72677	.69946	.71468
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223
36	.65077	.75927	.66393	.74780	.67688	.73610	.68963	.72417	.70215	.71203
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162
39	.65144	.75870	.66457	.74722	.67752	.73551	.69025	.72357	.70277	.71141
40	.65166	.75851	.66479	.74703	.67773	.73531	.69046	.72337	.70298	.71121
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019
46	.65298	.75737	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998
47	.65320	.75718	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine
	49°		48°		47°		46°		45°	

	0°		1°		2°		3°		
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00089	8437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00168	1718.87	.01804	55.4415	.03550	28.1684	.05299	18.8711	58
3	.00247	1145.92	.01833	54.5618	.03579	27.9372	.05328	18.7678	57
4	.00316	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00385	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00454	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00523	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00592	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00661	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00730	343.774	.02036	49.1089	.03783	26.4316	.05532	18.0750	50
11	.00799	312.521	.02065	48.4121	.03812	26.2296	.05561	17.9802	49
12	.00868	286.478	.02094	47.7305	.03841	26.0307	.05590	17.8863	48
13	.00937	264.441	.02123	47.0653	.03870	25.8348	.05619	17.7934	47
14	.01006	245.552	.02152	46.4189	.03900	25.6418	.05648	17.7015	46
15	.01075	229.182	.02181	45.7924	.03929	25.4517	.05677	17.6106	45
16	.01144	214.858	.02210	45.2861	.03958	25.2644	.05706	17.5205	44
17	.01213	202.219	.02239	44.8036	.03987	25.0798	.05735	17.4314	43
18	.01282	190.984	.02268	44.3461	.04016	24.8978	.05764	17.3432	42
19	.01351	180.932	.02297	43.9081	.04045	24.7185	.05793	17.2558	41
20	.01420	171.885	.02326	43.4941	.04074	24.5418	.05822	17.1698	40
21	.01489	163.700	.02355	43.1035	.04103	24.3675	.05851	17.0857	39
22	.01558	156.259	.02384	42.7358	.04132	24.1957	.05880	16.9990	38
23	.01627	149.465	.02413	42.3905	.04161	24.0263	.05909	16.9150	37
24	.01696	143.297	.02442	42.0674	.04190	23.8598	.05938	16.8319	36
25	.01765	137.507	.02471	41.7658	.04219	23.6965	.05967	16.7496	35
26	.01834	132.219	.02500	41.4851	.04248	23.5321	.05996	16.6681	34
27	.01903	127.321	.02529	41.2248	.04277	23.3718	.06025	16.5874	33
28	.01972	122.774	.02558	40.9845	.04306	23.2137	.06054	16.5075	32
29	.02041	118.540	.02587	40.7637	.04335	23.0577	.06083	16.4283	31
30	.02110	114.589	.02616	40.5619	.04364	22.9038	.06112	16.3499	30
31	.02179	110.892	.02645	40.3786	.04393	22.7519	.06141	16.2722	29
32	.02248	107.426	.02674	40.2137	.04422	22.6020	.06170	16.1952	28
33	.02317	104.171	.02703	40.0660	.04451	22.4541	.06199	16.1190	27
34	.02386	101.107	.02732	39.9347	.04480	22.3081	.06228	16.0436	26
35	.02455	98.2179	.02761	39.8176	.04509	22.1640	.06257	15.9687	25
36	.02524	95.4895	.02790	39.7145	.04538	22.0217	.06286	15.8945	24
37	.02593	92.9085	.02819	39.6243	.04567	21.8813	.06315	15.8211	23
38	.02662	90.4683	.02848	39.5465	.04596	21.7428	.06344	15.7483	22
39	.02731	88.1436	.02877	39.4801	.04625	21.6056	.06373	15.6763	21
40	.02800	85.9398	.02906	39.4248	.04654	21.4704	.06402	15.6043	20
41	.02869	83.8435	.02935	39.3805	.04683	21.3369	.06431	15.5340	19
42	.02938	81.8470	.02964	39.3468	.04712	21.2049	.06460	15.4638	18
43	.03007	79.9434	.02993	39.3233	.04741	21.0747	.06489	15.3943	17
44	.03076	78.1263	.03022	39.3003	.04770	20.9460	.06518	15.3254	16
45	.03145	76.3900	.03051	39.2778	.04799	20.8188	.06547	15.2571	15
46	.03214	74.7322	.03080	39.2648	.04828	20.6933	.06576	15.1893	14
47	.03283	73.1390	.03109	39.2511	.04857	20.5691	.06605	15.1222	13
48	.03352	71.6151	.03138	39.2365	.04886	20.4465	.06634	15.0557	12
49	.03421	70.1533	.03167	39.2219	.04915	20.3253	.06663	14.9898	11
50	.03490	68.7501	.03196	39.2071	.04944	20.2056	.06692	14.9244	10
51	.03559	67.4019	.03225	39.1920	.04973	20.0873	.06721	14.8596	9
52	.03628	66.1055	.03254	39.1768	.05002	19.9702	.06750	14.7954	8
53	.03697	64.8580	.03283	39.1616	.05031	19.8546	.06779	14.7317	7
54	.03766	63.6567	.03312	39.1464	.05060	19.7403	.06808	14.6686	6
55	.03835	62.4993	.03341	39.1312	.05089	19.6273	.06837	14.6059	5
56	.03904	61.3829	.03370	39.1160	.05118	19.5156	.06866	14.5438	4
57	.03973	60.3068	.03399	39.1008	.05147	19.4051	.06895	14.4823	3
58	.04042	59.2659	.03428	39.0856	.05176	19.2959	.06924	14.4213	2
59	.04111	58.2613	.03457	39.0704	.05205	19.1879	.06953	14.3607	1
60	.04180	57.2900	.03486	39.0552	.05234	19.0811	.06982	14.3007	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	89°		88°		87°		86°		

	4°		5°		6°		7°		
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.06998	14.3007	.08749	11.4301	.10510	9.51436	.12378	8.14485	60
1	.07023	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10386	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04755	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02846	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09043	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9883	.10863	9.20516	.12633	7.91583	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9963	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09483	10.5463	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5138	.11276	8.86862	.13047	7.66466	34
27	.07783	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7063	.09630	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07900	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11453	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11483	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0773	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	85°		84°		83°		82°		

	8°		9°		10°		11°		
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.14054	7.11537	.15898	6.31375	.17633	5.67128	.19436	5.14455	60
1	.14084	7.10098	.15968	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15998	6.29007	.17693	5.65305	.19498	5.12862	58
3	.14143	7.07059	.16028	6.27929	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.16058	6.26855	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.16088	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16117	6.24321	.17813	5.61392	.19619	5.09704	54
7	.14262	6.91174	.16147	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16177	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19708	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51178	.19951	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19981	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66468	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15273	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	81°		80°		79°		78°		

	12°		13°		14°		15°		
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.21256	4.70468	.23087	4.38148	.24933	4.01078	.26795	3.73805	60
1	.21286	4.69791	.23117	4.38573	.24964	4.00588	.26826	3.73771	59
2	.21316	4.69121	.23148	4.39001	.24995	4.00098	.26857	3.73738	58
3	.21347	4.68452	.23179	4.39430	.25026	3.99608	.26888	3.73707	57
4	.21377	4.67786	.23209	4.39860	.25056	3.99109	.26920	3.73676	56
5	.21408	4.67121	.23240	4.40291	.25087	3.98607	.26951	3.73646	55
6	.21438	4.66458	.23271	4.40724	.25118	3.98117	.26982	3.73616	54
7	.21469	4.65797	.23301	4.41159	.25149	3.97627	.27013	3.73586	53
8	.21499	4.65138	.23332	4.41595	.25180	3.97139	.27044	3.73557	52
9	.21529	4.64480	.23363	4.42032	.25211	3.96651	.27075	3.73528	51
10	.21560	4.63825	.23393	4.42471	.25242	3.96165	.27107	3.73500	50
11	.21590	4.63171	.23424	4.42911	.25273	3.95680	.27138	3.73472	49
12	.21621	4.62518	.23455	4.43352	.25304	3.95196	.27169	3.73444	48
13	.21651	4.61866	.23485	4.43795	.25335	3.94713	.27201	3.73416	47
14	.21682	4.61219	.23516	4.44239	.25366	3.94232	.27232	3.73388	46
15	.21712	4.60572	.23547	4.44685	.25397	3.93751	.27263	3.73360	45
16	.21743	4.59927	.23578	4.45132	.25428	3.93271	.27294	3.73332	44
17	.21773	4.59283	.23608	4.45580	.25459	3.92793	.27325	3.73304	43
18	.21804	4.58641	.23639	4.46030	.25490	3.92316	.27357	3.73276	42
19	.21834	4.58001	.23670	4.46481	.25521	3.91839	.27388	3.73248	41
20	.21864	4.57363	.23700	4.46933	.25552	3.91364	.27419	3.73220	40
21	.21895	4.56726	.23731	4.47387	.25583	3.90890	.27451	3.73192	39
22	.21925	4.56091	.23762	4.47842	.25614	3.90417	.27482	3.73164	38
23	.21956	4.55458	.23793	4.48298	.25645	3.89945	.27513	3.73136	37
24	.21986	4.54826	.23823	4.48756	.25676	3.89474	.27545	3.73108	36
25	.22017	4.54196	.23854	4.49215	.25707	3.89004	.27576	3.73080	35
26	.22047	4.53568	.23885	4.49675	.25738	3.88535	.27607	3.73052	34
27	.22078	4.52941	.23916	4.50137	.25769	3.88068	.27638	3.73024	33
28	.22108	4.52316	.23946	4.50600	.25800	3.87601	.27670	3.72996	32
29	.22139	4.51693	.23977	4.51064	.25831	3.87136	.27701	3.72968	31
30	.22169	4.51071	.24008	4.51530	.25862	3.86671	.27732	3.72940	30
31	.22200	4.50451	.24039	4.51997	.25893	3.86206	.27764	3.72912	29
32	.22231	4.49832	.24069	4.52465	.25924	3.85745	.27795	3.72884	28
33	.22261	4.49215	.24100	4.52934	.25955	3.85284	.27826	3.72856	27
34	.22292	4.48600	.24131	4.53405	.25986	3.84824	.27858	3.72828	26
35	.22323	4.47986	.24162	4.53877	.26017	3.84364	.27889	3.72800	25
36	.22353	4.47374	.24193	4.54350	.26048	3.83906	.27921	3.72772	24
37	.22383	4.46764	.24223	4.54825	.26079	3.83449	.27952	3.72744	23
38	.22414	4.46155	.24254	4.55301	.26110	3.82992	.27983	3.72716	22
39	.22444	4.45548	.24285	4.55778	.26141	3.82537	.28015	3.72688	21
40	.22475	4.44942	.24316	4.56256	.26172	3.82083	.28046	3.72660	20
41	.22505	4.44338	.24347	4.56736	.26203	3.81630	.28077	3.72632	19
42	.22536	4.43735	.24377	4.57216	.26235	3.81177	.28109	3.72604	18
43	.22567	4.43134	.24408	4.57699	.26266	3.80726	.28140	3.72576	17
44	.22597	4.42534	.24439	4.58182	.26297	3.80276	.28172	3.72548	16
45	.22628	4.41936	.24470	4.58666	.26328	3.79827	.28203	3.72520	15
46	.22658	4.41340	.24501	4.59152	.26359	3.79378	.28234	3.72492	14
47	.22689	4.40745	.24532	4.59639	.26390	3.78931	.28266	3.72464	13
48	.22719	4.40152	.24562	4.60127	.26421	3.78485	.28297	3.72436	12
49	.22750	4.39560	.24593	4.60616	.26453	3.78040	.28329	3.72408	11
50	.22781	4.38969	.24624	4.61107	.26483	3.77595	.28360	3.72380	10
51	.22811	4.38381	.24655	4.61599	.26515	3.77153	.28391	3.72352	9
52	.22842	4.37793	.24686	4.62092	.26546	3.76709	.28423	3.72324	8
53	.22873	4.37207	.24717	4.62586	.26577	3.76268	.28454	3.72296	7
54	.22903	4.36623	.24747	4.63081	.26608	3.75828	.28486	3.72268	6
55	.22934	4.36040	.24778	4.63578	.26639	3.75388	.28517	3.72240	5
56	.22964	4.35459	.24809	4.64076	.26670	3.74950	.28549	3.72212	4
57	.22995	4.34879	.24840	4.64574	.26701	3.74513	.28580	3.72184	3
58	.23026	4.34300	.24871	4.65074	.26733	3.74075	.28612	3.72156	2
59	.23056	4.33723	.24902	4.65576	.26764	3.73640	.28643	3.72128	1
60	.23087	4.33148	.24933	4.66078	.26795	3.73205	.28675	3.72100	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	77°		76°		75°		74°		

	16°		17°		18°		19°		
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.26676	3.48741	.30673	3.27086	.32492	3.07768	.34438	2.90431	60
1	.26706	3.48599	.30605	3.26745	.32534	3.07464	.34466	2.90147	59
2	.26736	3.47977	.30637	3.26406	.32566	3.07160	.34493	2.89873	58
3	.26769	3.47596	.30669	3.26067	.32598	3.06857	.34520	2.89600	57
4	.26800	3.47216	.30700	3.25730	.32631	3.06554	.34548	2.89327	56
5	.26832	3.46837	.30732	3.25393	.32663	3.06252	.34576	2.89055	55
6	.26864	3.46458	.30764	3.25055	.32695	3.05950	.34603	2.88783	54
7	.26896	3.46080	.30796	3.24719	.32727	3.05649	.34631	2.88511	53
8	.26927	3.45702	.30828	3.24383	.32759	3.05349	.34658	2.88240	52
9	.26958	3.45327	.30860	3.24049	.32792	3.05049	.34726	2.87970	51
10	.26990	3.44951	.30891	3.23714	.32824	3.04749	.34758	2.87700	50
11	.27021	3.44576	.30923	3.23381	.32856	3.04450	.34791	2.87430	49
12	.27053	3.44202	.30955	3.23048	.32888	3.04152	.34824	2.87161	48
13	.27084	3.43829	.30987	3.22715	.32921	3.03854	.34856	2.86892	47
14	.27116	3.43456	.31019	3.22384	.32953	3.03556	.34889	2.86624	46
15	.27147	3.43084	.31051	3.22053	.32985	3.03259	.34922	2.86356	45
16	.27179	3.42718	.31083	3.21722	.33017	3.02963	.34954	2.86089	44
17	.27210	3.42343	.31115	3.21392	.33049	3.02667	.34987	2.85822	43
18	.27242	3.41973	.31147	3.21063	.33082	3.02372	.35020	2.85555	42
19	.27274	3.41604	.31178	3.20734	.33114	3.02077	.35052	2.85289	41
20	.27306	3.41236	.31210	3.20406	.33146	3.01783	.35085	2.85023	40
21	.27337	3.40869	.31242	3.20079	.33179	3.01489	.35118	2.84758	39
22	.27369	3.40502	.31274	3.19752	.33211	3.01196	.35150	2.84494	38
23	.27400	3.40136	.31306	3.19426	.33243	3.00903	.35183	2.84229	37
24	.27432	3.39771	.31338	3.19100	.33276	3.00611	.35216	2.83965	36
25	.27463	3.39406	.31370	3.18775	.33308	3.00319	.35248	2.83702	35
26	.27495	3.39042	.31402	3.18451	.33340	3.00028	.35281	2.83439	34
27	.27526	3.38679	.31434	3.18127	.33373	2.99738	.35314	2.83176	33
28	.27558	3.38317	.31466	3.17804	.33405	2.99447	.35346	2.82914	32
29	.27590	3.37955	.31498	3.17481	.33437	2.99158	.35379	2.82653	31
30	.27621	3.37594	.31530	3.17159	.33469	2.98868	.35412	2.82391	30
31	.27653	3.37234	.31562	3.16838	.33502	2.98580	.35445	2.82130	29
32	.27685	3.36875	.31594	3.16517	.33534	2.98292	.35477	2.81870	28
33	.27716	3.36516	.31626	3.16197	.33567	2.98004	.35510	2.81610	27
34	.27748	3.36158	.31658	3.15877	.33600	2.97717	.35543	2.81350	26
35	.27780	3.35800	.31690	3.15558	.33632	2.97430	.35576	2.81091	25
36	.27811	3.35443	.31722	3.15240	.33665	2.97144	.35608	2.80833	24
37	.27843	3.35087	.31754	3.14923	.33698	2.96858	.35641	2.80574	23
38	.27875	3.34732	.31786	3.14608	.33731	2.96573	.35674	2.80316	22
39	.27906	3.34377	.31818	3.14288	.33763	2.96288	.35707	2.80059	21
40	.27938	3.34023	.31850	3.13973	.33796	2.96004	.35740	2.79802	20
41	.27970	3.33670	.31882	3.13656	.33828	2.95721	.35772	2.79545	19
42	.28001	3.33317	.31914	3.13341	.33861	2.95437	.35805	2.79289	18
43	.28033	3.32965	.31946	3.13027	.33893	2.95155	.35838	2.79033	17
44	.28065	3.32614	.31978	3.12713	.33926	2.94873	.35871	2.78778	16
45	.28097	3.32264	.32010	3.12400	.33958	2.94591	.35904	2.78523	15
46	.28128	3.31914	.32042	3.12087	.33991	2.94309	.35937	2.78269	14
47	.28160	3.31565	.32074	3.11775	.34023	2.94028	.35969	2.78014	13
48	.28192	3.31216	.32106	3.11464	.34056	2.93748	.36002	2.77761	12
49	.28224	3.30868	.32138	3.11153	.34088	2.93468	.36035	2.77507	11
50	.28255	3.30521	.32171	3.10843	.34121	2.93189	.36068	2.77254	10
51	.28287	3.30174	.32203	3.10533	.34153	2.92910	.36101	2.77002	9
52	.28319	3.29829	.32235	3.10223	.34186	2.92633	.36134	2.76750	8
53	.28351	3.29484	.32267	3.09914	.34218	2.92354	.36167	2.76498	7
54	.28383	3.29139	.32299	3.09606	.34251	2.92076	.36199	2.76247	6
55	.28414	3.28795	.32331	3.09298	.34283	2.91799	.36232	2.75996	5
56	.28446	3.28452	.32363	3.08991	.34316	2.91523	.36265	2.75746	4
57	.28478	3.28109	.32396	3.08685	.34348	2.91246	.36298	2.75496	3
58	.28509	3.27767	.32428	3.08379	.34381	2.90971	.36331	2.75246	2
59	.28541	3.27426	.32460	3.08073	.34413	2.90696	.36364	2.74997	1
60	.28573	3.27086	.32492	3.07768	.34446	2.90421	.36397	2.74748	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	73°		72°		71°		70°		

	20°		21°		22°		23°		
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34635	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72525	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43423	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67699	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29618	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39863	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	69°		68°		67°		66°		

	24°		25°		26°		27°		
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.44523	2.24604	.46631	2.14451	.48773	2.05080	.50953	1.92821	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.92610	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.92397	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.92188	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.91976	56
5	.44697	2.23727	.46806	2.13639	.48953	2.04276	.51136	1.91765	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.91551	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.91337	53
8	.44802	2.23204	.46914	2.13154	.49063	2.03825	.51246	1.91123	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.90907	51
10	.44873	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.90693	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.90478	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.90263	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.90048	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.90001	46
15	.45047	2.21993	.47163	2.12030	.49315	2.02780	.51503	1.90000	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.90000	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.90000	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.90000	42
19	.45187	2.21304	.47305	2.11393	.49459	2.02187	.51651	1.90000	41
20	.45222	2.21133	.47341	2.11233	.49495	2.02039	.51688	1.90000	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.90000	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.90000	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.90000	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.90000	36
25	.45397	2.20278	.47519	2.10443	.49677	2.01302	.51872	1.90000	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.90000	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.90000	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.90000	32
29	.45538	2.19599	.47663	2.09811	.49822	2.00715	.52020	1.90000	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.90000	30
31	.45608	2.19261	.47733	2.09496	.49894	2.00423	.52094	1.91928	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08873	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99696	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99404	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99259	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17583	.48091	2.07939	.50258	1.98973	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90473	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48522	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88600	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	65°		64°		63°		62°		

	28°		29°		30°		31°		
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60066	1.66428	63
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60185	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60245	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60304	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60364	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60424	1.65772	54
7	.53433	1.87152	.55697	1.79543	.58007	1.72393	.60484	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60543	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60603	1.65445	51
10	.53545	1.86760	.55813	1.79174	.58124	1.72047	.60663	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58163	1.71932	.60723	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60783	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60843	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71587	.60903	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60963	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.61023	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.61083	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.61143	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.61203	1.64363	41
20	.53920	1.85463	.56194	1.77955	.58513	1.70901	.61263	1.64255	40
21	.53957	1.85333	.56232	1.77834	.58553	1.70787	.61323	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.61383	1.64041	38
23	.54033	1.85075	.56309	1.77592	.58631	1.70560	.61443	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61503	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61563	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61623	1.63612	34
27	.54183	1.84561	.56463	1.77110	.58787	1.70106	.61683	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61743	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61803	1.63291	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61863	1.63183	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61923	1.63076	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61983	1.62969	28
33	.54409	1.83794	.56693	1.76390	.59023	1.69428	.62043	1.62862	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.62103	1.62755	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.62163	1.62648	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.62223	1.62541	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.62283	1.62434	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.62343	1.62326	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.62403	1.62219	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.62463	1.62112	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.62523	1.62005	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.62583	1.61898	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.62643	1.61791	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.62703	1.61684	16
45	.54862	1.82277	.57155	1.74964	.59494	1.68085	.62763	1.61577	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.62823	1.61470	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.62883	1.61363	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62943	1.61256	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.63003	1.61149	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.63063	1.61042	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.63123	1.60935	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.63183	1.60828	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.63243	1.60721	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.63303	1.60614	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66977	.63363	1.60507	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.63423	1.60400	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.63483	1.60293	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.63543	1.60186	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.63603	1.60079	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.63663	1.60000	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	61°		60°		59°		58°		

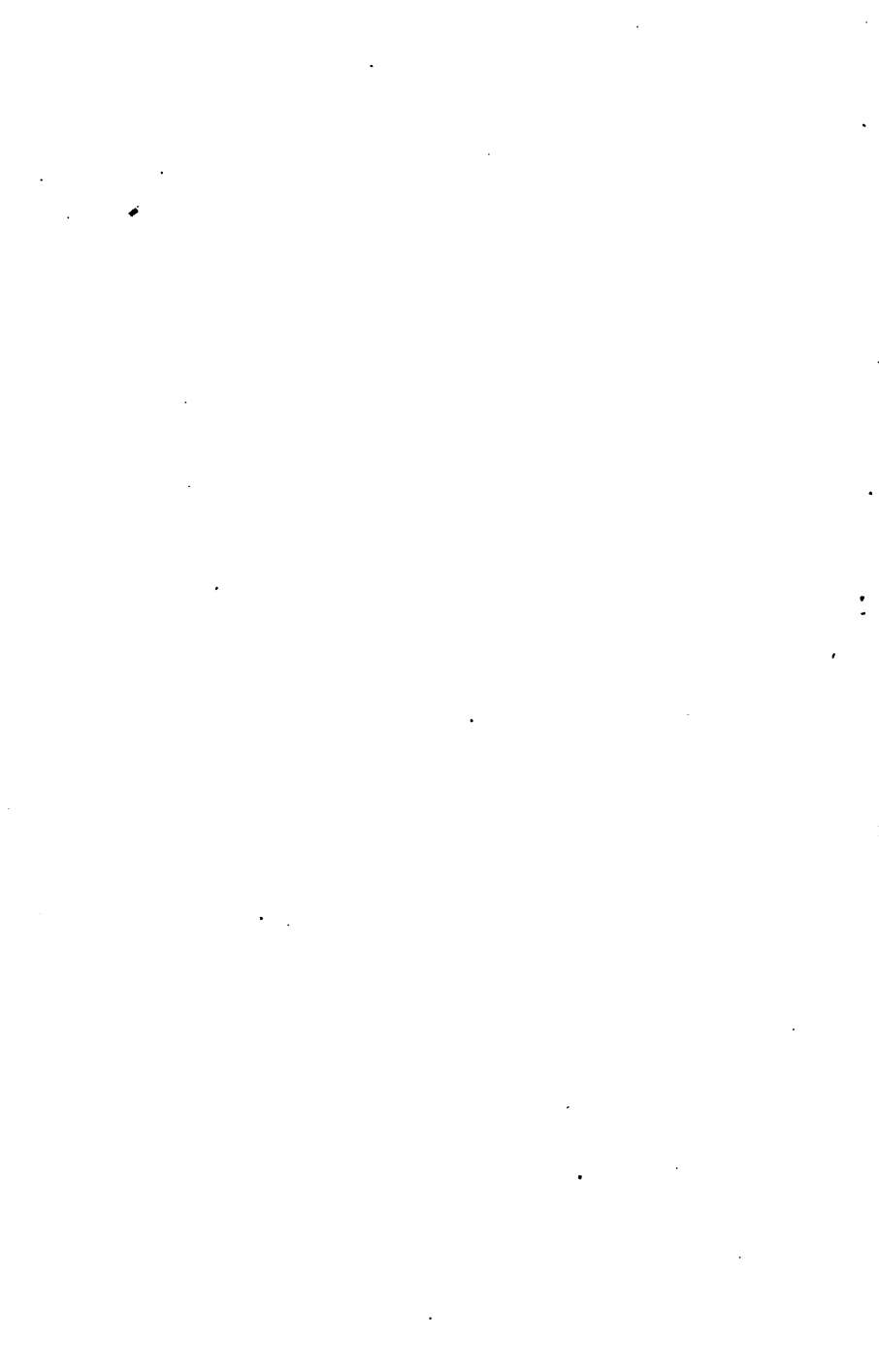
	32°		33°		34°		35°		
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.62487	1.60088	.64941	1.58986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59980	.64982	1.58898	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59872	.65024	1.58791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.58698	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.58595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.58497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.58400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.58302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.58205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.58107	.67832	1.47422	.70412	1.42022	51
10	.62893	1.59002	.65355	1.58010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.57913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.57816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.57719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.57622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.57525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.57429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.57332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.57235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.57138	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.57043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.56946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57777	.65854	1.56850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.56754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57574	.65938	1.56658	.68471	1.46046	.71066	1.40713	36
25	.63503	1.57473	.65980	1.56562	.68514	1.45955	.71110	1.40626	35
26	.63544	1.57372	.66021	1.56466	.68557	1.45864	.71154	1.40539	34
27	.63584	1.57271	.66063	1.56370	.68600	1.45773	.71198	1.40452	33
28	.63625	1.57170	.66105	1.56275	.68642	1.45682	.71242	1.40365	32
29	.63666	1.57069	.66147	1.56179	.68685	1.45592	.71285	1.40278	31
30	.63707	1.56968	.66189	1.56084	.68728	1.45501	.71329	1.40191	30
31	.63748	1.56868	.66230	1.55988	.68771	1.45410	.71373	1.40104	29
32	.63789	1.56767	.66272	1.55893	.68814	1.45320	.71417	1.40017	28
33	.63830	1.56667	.66314	1.55797	.68857	1.45229	.71461	1.39930	27
34	.63871	1.56566	.66356	1.55702	.68900	1.45139	.71505	1.39843	26
35	.63912	1.56466	.66398	1.55607	.68943	1.45049	.71549	1.39756	25
36	.63953	1.56366	.66440	1.55512	.68986	1.44958	.71593	1.39669	24
37	.63994	1.56265	.66482	1.55417	.69028	1.44868	.71637	1.39582	23
38	.64035	1.56165	.66524	1.55322	.69071	1.44778	.71681	1.39495	22
39	.64076	1.56065	.66566	1.55228	.69114	1.44688	.71725	1.39408	21
40	.64117	1.55966	.66608	1.55133	.69157	1.44598	.71769	1.39321	20
41	.64158	1.55866	.66650	1.55038	.69200	1.44508	.71813	1.39234	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39147	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39060	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38973	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38886	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38800	14
47	.64404	1.55269	.66902	1.49473	.69459	1.43970	.72078	1.38713	13
48	.64445	1.55170	.66944	1.49380	.69503	1.43881	.72122	1.38626	12
49	.64487	1.55071	.66986	1.49288	.69545	1.43792	.72167	1.38539	11
50	.64528	1.54973	.67028	1.49190	.69588	1.43703	.72211	1.38452	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38365	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38278	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38191	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38104	6
55	.64734	1.54478	.67239	1.48723	.69804	1.43258	.72433	1.38017	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37930	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37843	3
58	.64858	1.54183	.67366	1.48443	.69934	1.42992	.72565	1.37756	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37669	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37582	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	57°		56°		55°		54°		

	36°		37°		38°		39°		
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.72654	1.37698	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72833	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30953	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21743	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25643	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20806	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79973	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75083	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83663	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	53°		52°		51°		50°		

	40°		41°		42°		43°		
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15087	.90040	1.11081	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10808	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10673	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93632	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93686	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87853	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06055	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86369	1.15783	.89463	1.11778	.92655	1.07927	.95953	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92871	1.07676	.96176	1.03975	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07363	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
	49°		48°		47°		46°		

44°				44°				44°			
	Tang	Cotang			Tang	Cotang			Tang	Cotang	
0	.96569	1.03553	60	30	.97700	1.02355	40	40	.98843	1.01170	30
1	.96625	1.03498	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03193	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03073	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01643	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
	Cotang	Tang			Cotang	Tang			Cotang	Tang	
	45°				45°				45°		











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Retaining-walls for earth.  
Cabot Science

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